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VIBRATION IN SPARK-BLOWN CLOSED QUILL TUBES. ELECTRIC OSCILLATION*

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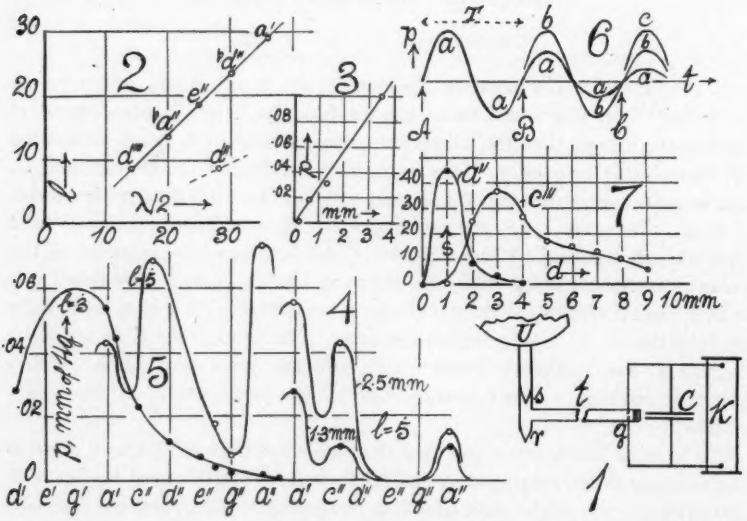
1. *Apparatus.*—The present experiments are in the main similar to the preceding** on the vibration of the air filament in quill tubes closed at both ends, except that the vibration is maintained by a spark succession of controllable frequency. The resonance conditions are determined by the acoustic pressures generated, and are measured by the interferometer U-tube. In figure 1, t is the quill tube 3.5 mm. in diameter, closed by a plug at one end and by the salient (s) and reëntrant (r) pinholes at the other, s communicating with one shank of the U-gauge. The spark gap is at g, sealed within the tube, C being a condenser with a thin loose mica plate between, K, a large induction coil. The primary of K is fed by a current at low voltage (1 storage cell) and the break may be set at any pitch at pleasure. Short sparks only (a few millimeters, at most) are wanted.

2. *Acoustic Vibrations.*—Barring the capriciousness of spark apparatus the acoustic experiments succeeded fairly well, as is evidenced by figure 2 Here the length l of the quill tube t, in centimeters, is mapped out in terms of the free semi-wavelength, $\lambda/2$. As far as the ear can discriminate, these data are reproduced by $l + 4.6 = 0.93\ (\lambda/2)$. Interpreting this as the first overtone, the confined wave-length $\lambda' + 4.6 = 0.46\lambda$, the coefficient 0.46 agreeing in order of value with the main results obtained when similar tubes (diameter .35 cm.) were actuated by the telephone. The strong d'' of figure 2 would be referred to the fundamental, as there suggested. Tubes less than $l = 5$ cm. in length, owing to irregularities of shape and section, no longer conformed to the locus of figure 2. The resonances, however (see figures 4 and 5), are still marked until the length of t is but 3 cm. (figure 5), when only the usual flat crests of very low frequencies are observed.

3. *Spark Length.*—Since the quill tube receives the energy per sec. con-

veyed by the spark, I was under the impression that the acoustic pressure would be independent of spark length. In figure 4 are two graphs for spark gaps of 1.3 and 2.5 mm., respectively. The graphs are alike, but the intensities (acoustic pressure in mm. of mercury) very different, in favor of the longer gap. In figure 5 is a similar graph with a still longer gap (size not recorded). Figure 3 gives some data for spark lengths (abscissas) and acoustic pressures, p, for a tube length of 13 cm. at the same resonance pitch. One may estimate that, with the present apparatus, something less than .03 mm. of mercury is the pressure resulting per mm. of spark gap. If, therefore, the acoustic pressure is to be taken as proportional to the potential difference, *caet. par.*, at the spark gap, these observations deserve more detailed inquiry.



4. *Electric Oscillation.*—The chief difficulty here will be the identification of a crest due to electric resonance, among the normal acoustic crests of the tube itself. Hence the short tube, $l = 3$ cm. (figure 5), was selected, which is ordinarily without crests above g''. However, figures 4 and 5 show that insignificant crests may be highly accentuated by increased potential difference. Furthermore, to change the frequency at the primary would be inadvisable, because of unavoidable irregularities at the break, by which intensities (p) are often greatly modified. Thus it appears that a fixed frequency should be established and adjustments for frequency made at the condenser C (figure 1) only.

Again it is obvious that the oscillations of the circuit KC alone may be reduced to acoustic pitch. The circuit gC, being of enormous frequency

by comparison, is acoustically absent. There remains the effect on KC, of the resistance at the spark gap g, which will have to be ignored for lack of data.

The plan of the method of experiment is then suggested by figure 6. Let a be the acoustic pressure in the lapse of time, produced by the damped electric oscillation of a given spark. It is obvious that the maximum effect or crest will result when the effects of succeeding sparks b, c are in phase with it. It is to be remembered, however, that all troughs, being also produced by sparks, are at once rectified. Hence if the acoustic period is T, the lowest electric period compatible with a crest would be $T/2$. A selection is made by the experiment among the period $T/2$, $2T/2$, $3T/2$, etc., of an admissible period. If there is thus reënforcement for phase differences, $T/2$, the maximum interference occurs for phase differences of $T/4$. The compound rectified wave cannot be annulled on interference, but it is far less oscillatory and its period approaching $T/4$ passes out of range as if it were annulled.

Data.—An example of results obtained in this way is given in figure 7, where the abscissas denote the distance apart of the plates (10 cm. diam.) of the condenser in mm. The ordinates are acoustic pressures in scale parts (about $20/10^6$ mm. of mercury each). In one case the break was set at a steady pitch c'''; in the other at a''. As in each curve there is no other change implied than the capacity of the condenser, and as the initial graph, figure 5, has no crests, the maxima of figure 7 must be attributed to electric oscillation, so far as I see. With an improved condenser both the a'' and the c''' crests were considerably sharpened. The a'' crest from its nearness to $d = 0$ would necessarily be more cusplike. With a closed condenser sparks are quenched.

The computation is tentative. If for the massive circuit, figure 1, we write $T = 2\pi\sqrt{LC}$ where C is the total capacity, then the capacity within the coil may be eliminated by finite differences, so that

$$L = \Delta T^2/4\pi^2\Delta C$$

Since the change of C is effected by lifting the condenser plates of area A, from the distance d to d' apart, $L = 9 \times 10^{\prime\prime} \Delta T^2/\pi A\, \Delta(1/d)$. For c''', $T^2 = 10^{-6} \times .92$ and for a'', $T^2 = 10^{-6} \times 1.29$; $A = 78$ cm.^{2} and from figure 7, $\Delta(1/d) = 4.7$; whence $L = 200$ Henrys, an apparently reasonable value, though it may be 4 times larger or $^4/_9$ times smaller, the harmonic which fits being selected by the experiment. Nevertheless from the difficulty of producing a uniform spark succession of high acoustic frequency it would be premature to insist on the trustworthiness of an otherwise interesting result, until such a method has been brought under full control. This I have not yet been able to do.

* Advance note from a Report to the Carnegie Inst. of Washington, D. C.

** These PROCEEDINGS, 9, 1923 (266).

NOTE ON ELECTRICALLY EXPLODED WIRES IN HIGH VACUUM

By Sinclair Smith

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About a year ago Wendt and Irion[1] reported the presence of helium in tubes in which tungsten had been electrically exploded by the method of Anderson.[2] They further reported an absence of hydrogen. The presence of helium is not surprising, for helium has been observed in the hot-spark, in which conditions are quite similar to conditions in the explosions. Winchester[3] has shown that the helium found in the hot-spark is merely occluded, and not a product of electrode disintegration. The behavior of hydrogen is somewhat different. Winchester found that even though sparking was continued until the electrodes were entirely used up, hydrogen continued to appear in his discharge tube. Hence, it is interesting to note that hydrogen does not appear in tubes in which wires have been exploded.

As there seemed to be some question as to the origin of the helium in the experiments of Wendt and Irion, Dr. Anderson of the Mount Wilson Observatory suggested that the writer repeat these experiments, and very kindly offered the use of his condenser. Two types of tubes were used. The first, shown in figure 1, was the ordinary H tube. The wire to be exploded was stretched between two slotted aluminum holders, which were then pinched together by means of a pair of sharp-nosed pliers. The tube was put on the pump and, after evacuation, sealed. The wire was then exploded in the usual manner. Immediately after the explosion, whenever possible, a discharge was passed through the capillary part of the tube and the gas examined spectroscopically. In general this examination was impossible, since after the explosion the tubes were usually found to be non-conducting to voltages up to about 60,000, the highest tried. The tubes which could be examined showed mercury lines, hydrogen lines, and the hydro-carbon spectrum ordinarily attributed to stop-cock grease, etc. No helium lines were found, and in no case was there more gas present than barely enough to pass a discharge. Whenever a discharge could be passed through a tube after the explosion, it was considered that the tube had not had a sufficiently high pre-explosion vacuum.

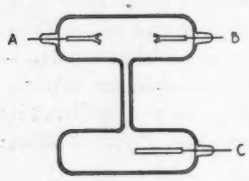

FIG. 1

In order to reduce the amount of metallic surface exposed in the explosion tube, a new tube was made, similar to the one described by Wendt and Irion, and shown in figure 2. All electrodes are of tungsten, and in

the case of the heavy electrodes A and B the seals run to the ends of the small rods which are ground flat and then drilled to a depth of about 1.5 mm. For loading, an opening was blown in the bulb, the wire sprung in place, and the opening sealed. After the bulb was evacuated the wire was exploded as before.

The chief difficulty encountered was in obtaining a sufficiently high vacuum in which to explode wires of tungsten. If the vacuum is not of the best, the discharge merely melts the wire in a few places, and arcs through the residual gas. Two diffusion pumps were used in series, backed by a good mechanical oil pump. During the entire pumping period the bulb was kept at about 350° C in order that it might be thoroughly "outgassed." A small tube of cocoanut charcoal, which was attached to the explosion chamber, was activated during this time. Liquid air traps were used in the usual manner. After from 20 to 24 hours' pumping the bulb and charcoal tube were sealed off and the charcoal tube was placed in liquid air for 20 minutes. After this the charcoal tube was in turn sealed off and the bulb was ready for the explosion.

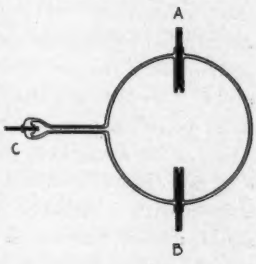

FIG. 2

A number of tungsten wires were exploded in tubes of each type and a few lead and aluminum wires were exploded in the H tubes. Both lead and aluminum formed bright mirrors in the nearby portions of the tube, while tungsten produced a band of dark stain on the glass. When the pre-explosion vacuum was good, none of the wires produced sufficient gas to pass a discharge. This is especially interesting in the case of aluminum, as this metal is ordinarily thought to contain much occluded gas. Possibly the gas is given off and then immediately reabsorbed by the thin film of aluminum on the walls of the tube.

[1] *J. Amer. Chem. Soc.*, **44**, 1922 (1987).
[2] *Astrophys. J.*, **51**, 1920 (37–48).
[3] *Phys. Rev.*, N. S., **3**, 1914 (287–294).

ON THE SUBDIVISION OF 3-SPACE BY A POLYHEDRON

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Communicated, November 12, 1923

It is known that a polyhedron in spherical 3-space subdivides the 3-space into two regions, but comparatively little has been determined, as yet, about the topological properties of the regions thus obtained. I shall prove that if the polyhedron be of genus zero it separates the 3-space into a pair of 3-cells, and that if it be of genus unity one, at least, of the regions must be tubular, as was predicted by Tietze.

Let α be a variable plane in a family of parallel planes $\{\alpha\}$, and let σ be any closed, bounded surface, without singularities, such that its intersection with α consists, in general, of a finite number of non-intersecting simple closed curves varying continuously with α. A finite number of exceptional positions of α will be permitted, in each of which one or the other of the following situations arises:

(I) The intersection of α with σ contains one *isolated point* P such that as the variable plane α passes in a properly chosen direction through the exceptional position, one of the curves of intersection shrinks to the point P and afterwards disappears.

(II) The curves of intersection have one *singular point* Q, which marks a regrouping of the branches of the curves meeting at Q. The system of curves through Q may always be thought of as composed of a finite number of simple curves with the point Q in common. If this number is $k + 1$, the point Q will be said to be of order k.

Evidently, a polyhedron belongs to the class of surfaces σ, but the planes $\{\alpha\}$ should be so oriented that no one of them passes through two vertices of the polyhedron.

Now, if a be the number of isolated points, b the sum of the multiplicities of the singular points, and p the genus of the surface σ, we shall have the following relations:

$$a - b = 2 - 2p$$
$$a \geq 2, b \geq 0, p \geq 0,$$

for the first relation is an immediate consequence of the Euler-Jonquières theorem on polyhedra, while the second expresses the fact that there are two extreme positions of the plane α for each of which the plane meets the surface σ in a single point.

Let us measure the degree of complexity of the surface σ by a number, n, equal to the number of simple curves in all the exceptional planes of the second type, including the curves which do not pass through the singular

points Q. First in order of simplicity will then be surfaces σ such that $n = 0$, and, consequently, $b = p = 0$, $a = 2$. Any surface of this sort will be swept out by a single simple closed curve which shrinks to a point when the plane of intersection α reaches its two extreme positions. By well-known theorems on logarithmic potential in two dimensions, it will be easy to set up coördinate systems within and without the surface σ in such a manner as to prove that the two sides of σ are 3-cells.

The case of a general surface σ of genus 0 will now be treated by induction with respect to n. In this connection, we shall use a fundamental combinatorial lemma to the effect that if two surfaces σ_1 and σ_2 of genus 0 meet in a 2-cell C and its boundary but in no other points, and if σ be the surface obtained by combining σ_1 with σ_2 and eliminating the cell C, then if σ_1 and σ_2 each subdivide the 3-space into a pair of 3-cells, so also must σ. The lemma offers no difficulties if it be understood that a 3-cell *with its boundary* is homeomorphic with the interior and surface of a sphere. In virtue of the lemma, all that will be needed to complete the induction will be to find the two surfaces σ_1 and σ_2 such that their complexities n_1 and n_2 are less than the complexity n of σ. This may be done as follows. Since n is no longer zero, there must be at least one exceptional plane α_0 of the second sort containing a singular point, Q_0. Moreover, there must be at least one curve of intersection k_0 in α_0 which encloses no other curve of intersection and which, therefore, bounds a 2-cell C in α_0 free from points of σ. Let us sever the surface σ into two simply connected pieces by a cut along the curve k_0 and let us then combine each of the pieces with the 2-cell C and its boundary to form a pair of surfaces σ_1 and σ_2 of genus 0. The surfaces σ_1 and σ_2 will not be of exactly the standard form to which we have limited the discussion, since each of them meets the plane α_0 in a 2-cell, but they may readily be brought to the required shape by a deformation. We shall distinguish two cases according as the singular point Q is off or on the curve k_0. If Q is off the curve, a plane α_1 of $\{\alpha\}$ adjacent to α_0 and on the proper side of α_0 will meet the surface σ_1 in a curve k_1 differing but little from the curve k_0 and approaching k_0 as α_1 approaches α_0. We shall modify the surface σ_1 by allowing the curve k_1 to shrink to a point as α_1 moves towards α_0, so that it disappears entirely before α_1 reaches α_0. Such a change in σ_1 will not affect the character of the regions bounded by this surface, as may easily be proved; nor will a similar change in σ_2 affect the regions bounded by σ_2. But since the effect of the two deformations will be to do away with k_0 as a curve of either σ_1 or σ_2, the numbers n_1 and n_2 of the deformed surfaces σ_1 and σ_2 will evidently be related to the number n of the original surface in the following manner:

$$n_1 + n_2 = n - 1,$$

whence,

$$n_1 < n \,.\, n_2 < n,$$

which completes the induction for this case. In the remaining case where the point Q is on the curve k_0, the only difference is that an arc, l_1, of a curve of intersection in α_1, and not necessarily an entire curve, approaches the curve k_0 as α_1 approaches α_0. The necessary deformation of σ_1 is one such that the arc (or curve) l_1 shrinks to the point Q as α_1 approaches α_0. We perform a similarly modified deformation on σ_2 and complete the argument just as before, thereby proving the theorem.

A similar reduction may be applied for the case $p = 1$, but at some stage of the process the curve k_0 will be non-bounding. The side of σ containing the plane surface C bounded by k_0 will thus have to be tubular, that is to say, homeomorphic with the interior of an anchor ring. This is the theorem predicted by Tietze. For a general value of p, it is easy to show that the linear connectivity of either region bounded by σ is $(P_1 - 1) = P$, but the group of the region may be very complicated.

AN EXAMPLE OF A SIMPLY CONNECTED SURFACE BOUNDING A REGION WHICH IS NOT SIMPLY CONNECTED

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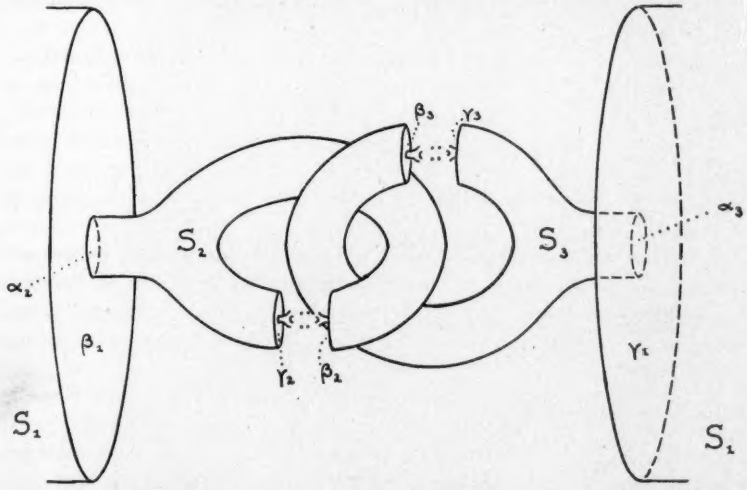
The following construction leads to a simplified example of a surface Σ of genus zero situated in spherical 3-space and such that its exterior is not a simply connected region. The surface Σ is obtained directly without the help of Antoine's inner limiting set.

The surface Σ will be the combination, modulo 2, of a denumerable infinity of simply connected surfaces S_i $(i = 1, 2, \ldots)$, all precisely similar in shape, though their dimensions diminish to zero as i increases without bound. The shape of the surface S_i may perhaps be described most readily by referring to the accompanying figure in which the surfaces S_2 and S_3 are represented. By comparison with S_2 to which, by hypothesis, all the other surfaces S_i are similar, we see that the general surface S_i is roughly like a tube twisted into the shape of the letter C and terminating in a pair of circular 2-cells, β_i and γ_i. There is, however, a slight protuberance in the side of the tube terminating in a 2-cell α_i.

The position of the surfaces S_1, S_2, and S_3 with respect to one another is indicated in the figure, though only the two ends of S_1 terminating in α_1 and β_1 are shown. It will be noticed that the faces α_2 of S_2 and α_3 of S_3 are subfaces of the faces β_1 and γ_1 of S_1, respectively, and that the surfaces

S_2 and S_3 are hooked around one another, so to speak. When S_2 and S_3 are added modulo 2 to S_1 (which means that the points of α_2 and α_3 must be deleted from the combined surfaces), a simple closed surface, Σ_1, is obtained. The surface Σ_1 will be regarded as the first approximation of the desired surface Σ. The next approximating surface, Σ_2, is obtained by adjoining to Σ_1, modulo 2, the next four surfaces, S_4, S_5, S_6, S_7. The first two of these will be related to S_2 and the last two to S_3 in exactly the same way that the surfaces S_2 and S_3 are related to S_1; that is to say, a similarity transformation of the 3-space carrying S_1 into S_2 would carry S_2 and S_3 into S_4 and S_5, respectively, while one carrying S_1 into S_3 would carry S_2 and S_3 into S_6 and S_7, respectively. The third approximation Σ_3 is obtained by adjoining the next eight surfaces S_8, ..., S_{15} in a similar manner, so that the pair S_8 and S_9 are attached to S_4 just as the pair S_2 and S_3 are



attached to S_1, and so on. The surface Σ is the limiting surface approached by the sequence Σ_1, Σ_2, Σ_3, .. It will be seen without difficulty that the interior of the limiting surface Σ is simply connected, and that the surface itself is of genus zero and without singularities, though a hasty glance at the surface might lead one to doubt this last statement. The exterior R of Σ is not simply connected, however, for a simple closed curve in R differing but little from the boundary of one of the cells γ_i cannot be deformed to a point within R. It is easily shown, in fact, that the group of R requires an infinite number of generators.

The points K of Σ which are not points of the approximating surfaces Σ_i form an inner limiting set of a much simpler type than the inner limiting set of Antoine, as was pointed out to me by Professor Veblen. For we

may close down upon the points K by a system of spheres rather than by a complicated system of linking anchor rings.

This example shows that a proof of the generalized Schönfliess theorem announced by me two years ago, but never published, is erroneous.

REMARKS ON A POINT SET CONSTRUCTED BY ANTOINE

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Communicated, November 16, 1923

From the consideration of a remarkable point set discovered by Antoine, the following two theorems may be derived:

Theorem 1. There exists a simple closed surface of genus 0 in 3-space such that the interior of the surface is not simply connected but has, on the contrary, an infinite group.

Theorem 2. There exists a simple closed curve in 3-space which is not knotted, inasmuch as it bounds a 2-cell without singularities, and yet such that its group[1] (as defined by Dehn) is not the same as the group of a circle in 3-space.

It follows without difficulty from the second theorem that if an isotopic deformation be defined in the customary manner (cf. for example, Veblen's Cambridge Colloquium Lectures), the group of a curve in 3-space is not an isotopic invariant. This suggests that a modified definition of isotopy might be advisable.

Antoine's point set is obtainable as follows. Within an anchor ring π in 3-space, we first construct a chain C of anchor rings π_i ($i = 1, 2, \ldots, s$) such that each ring π_i is linked with its immediate predecessor and immediate successor, after the manner of links in an ordinary chain, and such, also, that the last ring π_s is linked with the first π_1, thereby making the chain closed. We further suppose that the chain C is constructed in such a way that it winds once around the axis of the ring π. Secondly, we make a similar construction within each of the anchor rings π_i, thereby obtaining chains C_i made up of rings π_{ij} ($j = 1, 2, \ldots, s$), and repeat the process indefinitely, obtaining chains C_{ij} within π_{ij}, C_{ijk} within π_{ijk}, and so on. If we think of the system of rings within one of the rings π_i as the image of the system of rings within the ring π under a similarity transformation carrying the interior and boundary of π into the interior and boundary of π_i, it is clear that the diameters of the rings $\pi_{ijk}\ldots$ decrease towards zero as the number of subscripts to their symbols increases. The inner limiting set Σ determined by the infinite sequences of rings $\pi_i, \pi_{ii}, \pi_{ijk}, \ldots$ is the

set constructed by Antoine. It has the remarkable property, as was pointed out by its discoverer, of being totally disconnected, since every ring $\pi_{ijk}\ldots$ separates it into two non-overlapping closed set, but such that every simple closed surface of genus O which contains a point of Σ in its interior and a point of Σ in its exterior necessarily meets the set in at least one point. Let me also call attention to the following property which is the key to the present discussion: the group of the region R residual to the set is infinite and requires an infinite number of generators for its definition. This can easily be shown by noticing that the generating circle of any one of the anchor rings $\pi_{ijk}\ldots$ links the point set Σ in a manner characteristic to itself which cannot be reproduced by any combination of generating circles of any of the other rings. Therefore, if these generating circles be all deformed in the space exterior to Σ so as to pass through a common point, P, they will determine an infinite number of independent generators of the group.

Following Antoine,[2] let us now construct a simply connected region in the space R residual to Σ as follows: We first take a 3-cell E in the space exterior to the ring π but such that its boundary meets the boundary of the ring π in the points and boundary points of a 2-cell e. Secondly, in the region between the ring π and the rings π_i $(i = 1, 2, \ldots, s)$, we take s 3-cells E_i $(i = 1, 2, \ldots, s)$, such that no two of them have a point or boundary point in common, but such that the boundary of E_i meets the boundary of π in the points and boundary points of a 2-cell t_i interior to e, and the boundary of the ring π_i in the points and boundary points of a 2-cell e_i. The boundary of E_i does not, however, meet the boundaries of any of the rings π_j $(i \pm j)$. Then, if we combine the 3-cell E with the 3-cells E along the 2-cells t_i, we thereby obtain a 3-cell F_1 bounded by a surface of genus O. We now repeat a similar construction within each of the rings π_i, and so on indefinitely, obtaining successively the 3-cells F_1, F_2, ..., where F_i is obtained by combining F_{i-1} with the new 3-cells constructed at the i'th stage of the process. As is easily shown, the sequence of 3-cells F_1, F_2, ... approaches a definite limiting 3-cell F bounded by a simple closed surface Σ_0, of genus O, which surface contains the set Σ. Now, however, since the group of the region R residual to Antoine's point set Σ requires an infinite number of generators, all of which may be located in the space exterior to Σ_0, the group of the space R_0 exterior to Σ_0 must *a fortiori* require an infinite number of generators. Therefore, the exterior R_0 of Σ_0 cannot be simply connected. This proves the first theorem, since the interior and exterior of Σ_0 may be interchanged by a reflection. It may be remarked, in passing, that, by a theorem which I have proved elsewhere,[3] the Betti numbers of the interior and exterior of Σ_0 must nevertheless satisfy the relation $P_1 = 1$; that is to say, every curve in either of these regions is bounding, though it need not bound a 2-cell.

The second theorem follows from a theorem by Denjoy to the effect that through the nowhere dense perfect set Σ on the surface Σ_0, it must be possible to trace a Jordon curve, J, on Σ_0. Evidently, the group of the 3-dimensional region residual to J requires an infinite number of generators, yet the curve J bounds a 2-cell in the 3-space, namely, one of the two 3-cells into which the curve J subdivides the surface Σ_0.

[1] More properly, the group in question is the group of the portion of 3-space residual to the curve.

[2] *C. R. Paris Acad. Sci.*, **171**, 1920 (66).

[3] *Trans. Amer. Math. Soc.*, **23**, 1922 (333–349).

A STUDY OF THE INTERACTION OF HOST AND PARASITE: A REACTION PRODUCT IN INFECTIONS WITH TRYPANOSOMA LEWISI WHICH INHIBITS THE REPRODUCTION OF THE TRYPANOSOMES

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Trypanosoma lewisi, which is a non-pathogenic blood parasite occurring in various species of rats all over the world, offers many opportunities for studying the interaction of host and parasite. In our investigations of the various factors involved in this interaction, one of the most interesting results is the discovery of a specific reaction product, formed in the blood of infected rats, which forces upon the parasite a peculiar cycle of reproductive activity during the course of the infection. Before this reaction product can be described, it is necessary to discuss the general question of the resistance which the host offers against infections with *T. lewisi* and to outline the normal course of ordinary infections.

We use the term resistance (as in our earlier work) to denote collectively those factors, either active or passive, which operate adversely against the parasite and restrict ourselves to a consideration of only that resistance which develops after the parasites have successfully invaded the host. Such a resistance may be roughly measured by making daily determinations of the number of parasites. For example, if the parasites either remain constant or decrease in number, we know some type of resistance is operative. It may be brought about, however, by one or both of two factors viz.: (1) The rate of reproduction of the parasites may be retarded, or (2) the trypanosomes, after they are formed, may be destroyed. Thus: Number of parasites per cmm. of blood = Number produced by repro-

duction of parasites — Number destroyed in consequence of resistance.

To differentiate between these two factors, we have developed a method of measuring the rate of reproduction, and, hence, of getting the comparative number of parasites produced by reproduction irrespective of the number destroyed. (For details see Taliaferro, W. H. and L. G., *Amer. J. Hyg.*, **2,** 1922 (264–319).) Essentially, the method consists in comparing the coefficient of variation for some factor involving size (total length) of samples of trypanosomes taken at stated intervals throughout the course of an infection. The rationale of this method is based on the obvious and well-known fact that a sample taken, on one hand, from a population undergoing rapid reproduction, with the constant production of young forms and intermediate growth stages, will exhibit much greater variability in size than a sample, taken on the other hand, from a population in which there is little or no reproduction and in which all of the organisms are full-grown adults.

All of the following conclusions are based on determinations of the number of trypanosomes per cmm. of blood and of the coefficient of variation for total length of samples of trypanosomes. For a detailed description of the technique and methods of computation see Taliaferro, W. H. and L. G., *loc. cit.* The various coefficients of variation for rat 105 were each calculated from 100 specimens, those for the other infections from 50 specimens. In all cases the conclusions drawn from the coefficient of variation in regard to the rate of reproduction were checked by a microscopical examination of the slides for dividing forms.

The typical life-cycle of *T. lewisi* in the rat, which has been verified in detail in eight rats and in numerous other control rats, is well illustrated in rat 105. The trypanosomes first appeared in the blood (41,000 per cmm. of blood) four days after an intraperitoneal injection of a dilute suspension of infected blood. They increased in numbers (although at a progressively lower rate) until the tenth day (338,000) when they suddenly decreased (76,000 on the fourteenth day); after which they remained relatively constant until the thirty-sixth day when they disappeared altogether from the blood. The coefficient of variation for total length of the trypanosomes, on the other hand, was extremely high (25.32 per cent) when the trypanosomes first appeared in the blood but declined rapidly until the tenth day (3.95 per cent) after which it remained approximately at the level of what we have found to be the variability of non-reproducing "adult" trypanosomes, viz., 3.0%, throughout the rest of the infection. This indicates that the rate of reproduction was very high when the organisms first appeared in the blood, that it rapidly decreased until reproduction was completely inhibited by the tenth day and was never resumed. Direct examination of the slides also showed that dividing forms were extremely numerous on the first day after the trypanosomes appeared in

the blood, but that their number decreased until, after the tenth day, no dividing forms were encountered.

To sum up the results in this infection in terms of the effects of the resistance developed by the rat, we find: (1) As the result of an initial resistance, the rate of reproduction of the parasites was retarded more and more until it was finally inhibited altogether (by the tenth day). (2) A second effect of resistance developed between the tenth and fourteenth days when a large number of parasites were destroyed. (3) Finally, a third effect of resistance developed by the thirty-fifth day when all of the trypanosomes left in the blood were destroyed. As far as can be determined at the present time, these effects of resistance are due to different mechanisms.

In the present work we will limit ourselves to a consideration of the mechanism of the first effect of resistance, viz., the inhibition of reproduction. Our data indicate that this is due to the formation of a specific reaction product. Briefly, they show, if we inject a sufficient quantity of serum taken from the blood of a rat in which only non-reproducing "adult" trypanosomes exist (after the tenth day of the infection) together with some "adult" trypanosomes, into a fresh rat, that, instead of repeating the normal course of the infection, the trypanosomes simply exist in the blood without manifesting any reproductive activity.

The experimental procedure was as follows. A rat, designated as the *seed* rat, was injected *intraperitoneally* with a dilute suspension of infected blood, and, after the trypanosomes appeared in the blood, daily determinations of the coefficient of variation for total length were made until all reproductive activity had ceased, as indicated by a 3.0% coefficient of variation and the absence of any dividing forms. At this point the rat was etherized, its thoracic cavity opened, and as much blood as possible drawn from the heart under aseptic conditions and centrifuged. After centrifugalization, the tube containing the blood was gently rotated so that the trypanosomes which are deposited just above the leucocytic cream were stirred into the serum, and the serum containing the trypanosomes and some white and a very few red corpuscles was decanted and recentrifuged. The trypanosome-free serum was removed and the trypanosomes in the sediment washed three or four times in saline by separate centrifugalizations. At this point there was on the one hand trypanosome-free serum and on the other hand washed trypanosomes (with invariably a few white and red corpuscles) both of which came from the original seed rat. The final experimental procedure was to inject half of the washed trypanosomes, mixed with a specified dose of serum, *intravenously* into a non-infected rat (designated as the *experimental* rat) and the other half of the washed trypanosomes, mixed with a similar dose of normal serum (or physiological saline), in the same manner into a similar rat (designated as the *control* rat).

The following example is typical of the results obtained. Seed rat 972 was injected intraperitoneally, and, following the appearance of the organisms in the blood, after an incubation period of four days, daily determinations of the number of parasites per cmm. of blood and the coefficient of variation were made. The results are closely similar to those obtained in rat 105. By the tenth day, the trypanosomes had ceased all reproductive activity and were in the "adult" stage as evidenced by a coefficient of variation for total length of about 3.0% and the complete absence of dividing forms. The experimental procedure outlined above was then carried out with experimental rat 977 and control rat 980.

In the blood of the experimental rat 977, which received a dose of 2 cc. of serum per 100 grs. body weight plus the adult trypanosomes, both from the seed rat, the trypanosomes were found very quickly after injection and for the first eleven days were sufficiently numerous to study. Only an occasional organism was observed on the twelfth and thirteenth days and none thereafter. Throughout the entire infection, none of the trypanosomes exhibited any indications of reproduction; for the coefficient of variation remained between 2.0 and 3.0%, and daily microscopical examinations of the slides failed to reveal any evidences of reproduction or growth. This conclusion is also corroborated by the fact that the numbers remained between three and four thousand per cmm. of blood during the entire infection.

In marked contrast to the experimental rat, the infection in the control rat 980, which received a dose of 2 cc. of *normal* serum per 100 grs. body weight plus "adult" trypanosomes from the seed rat 972, pursued the normal course (c. f. rat 105), in that the coefficient of variation rose to 23.2% by the 5th day and gradually fell to 2.7% by the 12th day. Furthermore, the number of organisms, which was about 2000 per cmm. of blood on the first day of the infection, increased to the very high number of 613,000 per cmm. by the 13th day of the infection.

In other words, when the trypanosomes were injected into the blood stream of a clean rat with the addition of the serum containing the reaction product developed in infected rats, they were unable to reproduce (for at least thirteen days, which was as long as any of the experimental infections lasted), whereas, when trypanosomes from the same source were injected with the addition of a like quantity of normal serum, they began reproduction after two days and underwent the ordinary cycle of reproductive activity.

Various grades of inhibition were obtained by varying the dose of serum, such as (1) almost complete inhibition of reproduction where the trypanosomes in the experimental rat did not increase in number but where the coefficient of variation showed a slight rise (10.0%) about the sixth day of the infection, or (2) practically no inhibition of reproduction but simply

a delay in the reproductive activity as compared with the control. Furthermore, the titre of the serum for the reaction product increases during the course of the infection.

We have been unable to find any indication of the formation of any substance similar to or identical with the reaction product just described in *in vitro* cultures of *T. lewisi*.

Although we cannot say definitely that the present reaction product is different from the various immunological reactions known at the present time, the present work does emphasize a new factor or effect of the resistance of a host against an invading microörganism and explains the peculiar cycle of reproductive activity observable in *T. lewisi* in the rat. Furthermore, since it prevents the reproduction of the trypanosomes, it may play a very important part not only in the immunity of recovered rats to *T. lewisi* but also in the prevention of superinfections.

This work was carried out with the aid of Grant No. 235 of the BACHE FUND of the NATIONAL ACADEMY OF SCIENCES and will be presented in full in the *Journal of Experimental Medicine*.

RETARDED ESTABLISHMENT OF INTRODUCED PARASITES OF INJURIOUS INSECTS

BY L. O. HOWARD

BUREAU OF ENTOMOLOGY, U. S. DEPARTMENT OF AGRICULTURE

Read before the Academy, November 13, 1923

In international work with parasites of injurious insects it must be pointed out that we must not expect, and especially must we not predict speedy results with introduced forms. Speedy establishment we may hope for in many cases, but absolute control of the injurious form can be expected in the great majority of instances to come about only after a lapse of years if ever. In many cases, however, we may reasonably hope for a very considerable assistance from the introduced forms which will help greatly with other measures towards bringing about approximate control.

The following are some of the principal factors which act in delaying results:

(1) The necessity with many host insects for a sequence of parasites.

(2) The necessity for a rather prolonged period of time to elapse before the parasite, introduced in comparatively small numbers and even with a greater rate of multiplication than its host, can catch up in numbers to the millions of the host insects.

(3) The probable necessity for secondary hosts in the country to which the parasites are imported.

(4) The possibility of the interbreeding of the imported forms with native forms or races, thus producing offspring which will inherit the capacities of the native species only.

(5) Climatic conditions, either general or specific to a season, which, while not unfavorable to the host, may be very unfavorable to the introductions.

(6) In the case of Scoliids, the absence of flowering plants attractive to the adults.

There is much to be learned from a study of the work which has been done by the U. S. Bureau of Entomology and the State of Massachusetts with the parasites of the gipsy moth and the brown-tail moth. In fact, the laboratory at Melrose Highlands possesses a mass of notes, accumulated during the past nineteen years, which, when carefully studied, will throw much light upon many practical subjects as well as upon questions of great biological importance. Of the large number of species of parasites and predators which we have brought from all over Europe and from Japan, not more than half a dozen have succeeded definitely in becoming established and in spreading throughout the devastated area. For example, between 1906 and 1909 no less than eleven species of Tachniid parasites were imported in greater or less number and liberated for colonization in New England. Down to October 1923 not a single individual of any of these eleven species has been recovered; but to offset this we have the extraordinary success of *Compsilura concinnata* which has apparently received no check whatever in this country. It was liberated in 1906 and 1907 and has continued to spread and increase since that time. In the present year it is established all over New England and along the eastern border of New York State. This species, therefore, has acted just as we originally expected nearly all of them to act. It is curious to note, however, that even with this docile and valuable species our neighbors in Canada have had much more difficulty than we have had in Massachusetts. It was taken from Massachusetts and liberated in New Brunswick in 1911, but not until 1918 were Tothill and McLaine able definitely to state that the species had been established in Canada.

Practically the same thing has happened with the large series of Hymenopterous parasites imported. We have established several of them, but so far only two have done the work that was expected; but these two (both species of Apanteles) have more than justified all the efforts which the government and the State of Massachusetts have made in this direction.

The same thing may be said of that extraordinary Carabid beetle known as *Calosoma sycophanta* which we first imported in 1906. Down to 1911, after careful rearing in confinement, we had been able to liberate about eighteen thousand, adults and larvae. Since then the spread has been

extraordinary. The beetle and its larvae feed not only on the gipsy moth but on many of our other native caterpillars, and it is interesting to note that the species has spread far beyond the limits of spread of the gipsy moth. When the big colony of the gipsy moth was discovered in New Jersey on the Duke Place, the Calosoma had arrived there—how, no one knows—and was at work apparently before the gipsy moth itself had been identified by the State or government entomologists.

Some of the other species imported have been apparently lost, but have suddenly come to the front after a lapse of years and have multiplied in numbers.

Quite recently, and in another field, there have been two instances of delayed establishment which have greatly encouraged us. I refer to the recovery of *Scutellista cyanea* in Louisiana the past year and of *Pleurotropis epigonus* in Maryland, Pennsylvania and surrounding States during the past few years. The Scutellista was sent me by Berlese from Italy in 1898 and was put out by Prof. H. A. Morgan at Baton Rouge that same year. It was never recovered in Louisiana until last year, when it was found parasitizing the black scale in New Orleans. It is perfectly true that in the meantime the same species had been brought from South Africa to California and there established as an enemy of the black scale on citrus trees and on the pepper tree; and it is quite possible that both the scale and its contained parasite had been brought into Louisiana from California. But there remains in my mind a strong probability that it became established shortly after 1898, and that the reason possibly that it was not discovered was that it transferred its principal attention to another of its European hosts, namely, the black scale, whereas we were searching for it on the wax scale (Ceroplastes) on which it was originally liberated.

Pleurotropis epigonus, a parasite of the Hessian fly, was sent over to Professor Riley by Frederick Enock in 1890, and was liberated first by Webster at La Fayette, Indiana, by Forbes at Champaign, Ill., and by Cook at Agricultural College, Mich. In 1894 another small sending was received from Enock, and the parasites were released at Cecilton, Md., and Fredericktown, Md. Only a single specimen was found in 1895, and from that time on until 1917—twenty-two years—it was not recovered. Then it was turned up at Fredericktown, Md., by Mr. P. R. Myers of the Bureau of Entomology. The species has now become one of the common parasites of the Hessian fly throughout Maryland, Pennsylvania, New Jersey and New York.

Here we have a delayed recovery of twenty-five years in the one case and twenty-two years in the other. I deem it quite possible that perhaps a number of the parasites which we have brought over in former years for the gipsy moth, and for the recovery of which we have given up hope, may still be found and may still give us effective service.

DOES THE INHERITANCE OF DIFFERENCES IN GENERAL SIZE DEPEND UPON GENERAL OR SPECIAL SIZE FACTORS?

BY W. E. CASTLE

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Communicated, November 28, 1923

In a recent number of these PROCEEDINGS[1] Sumner criticizes adversely certain views which I had previously expressed concerning size inheritance in mammals, a subject which we have both been studying for some years, but on very different material, and this difference in material will perhaps account in part for the divergence between our conclusions. I have been studying size variations in races of domestic rabbits between which the size differences are very great, whereas he has been studying size variations in races of field mice between which the size differences are very small. My animals were of known age and ancestry, all adults and completely adapted to conditions of life in domestication; his animals were either wild caught individuals of unknown age and ancestry or when cage-born frequently showed diminished size and abnormalities indicating lack of complete adaptation to their environment. I do not suggest that my material was superior to his, for we were interested primarily in different questions, he in questions of racial evolution, I in the question of size inheritance. It is true that these questions are interrelated but each of us for his own special object of study had probably chosen his material wisely. Certainly for questions of inheritance it is necessary to have material genetically as pure as possible and to deal with two generations of offspring obtained by crossing races which differ markedly in the character to be studied. I maintain therefore that with reference to questions of inheritance my material is superior to his.

What, now, is the question at issue? Is total size the net result of independent variations (some plus, some minus) in the different parts of the body, or is it the result of general agencies which affect the entire body simultaneously in the same (plus or minus) direction? Sumner gives a qualified adhesion to the first of these alternatives and represents me as giving unqualified adhesion to the second. He declines to subscribe to Davenport's belief "that general factors control growth only to a degree that may be estimated as less than half," but does not substitute any other fraction for "less than half." "A considerable margin of independent variability" is the most precise statement to which he commits himself. I am represented as defending the view that size-factors are *exclusively general* in their action, and I must admit that there is some justification for this charge in an unfortunate sentence quoted by Sumner from page 19 of my paper[2] as follows: "I think that a strong probability has been

established that the genetic factors which affect size in mammals are general in their action, *exclusively* so." Insert before the word *size* in the foregoing sentence the word *total*, and the reader will get the meaning which I wished to convey. It did not occur to me at the time that I might be misunderstood and that the reader might think that I had in mind *any detail of size in any part of the body*. Such a position would be obviously untenable, not to say ridiculous. For twenty years ago I discovered in rabbits a unit-character affecting the length of the hair (known as the angora character) quite independent in its inheritance of general body size. Yet the angora condition might conceivably affect the total weight of the coat and so of the entire body, but to so slight an extent, that it is negligible as a factor in total body size. The same is true of the ear in sheep which is known to vary in a similar discontinuous way. Wriedt has shown that in sheep short-ear is an imperfectly dominant Mendelian character, which in a homozygous condition results in earlessness. Now here is an inherited character which affects the size of a part very fundamentally and may even cause it to disappear altogether and yet has no probable effect on general body size.

The same is true also of those slight quantitative variations in ear-length, foot-length and tail-length which Sumner has studied in mice. They have a negligible effect on total body size. They do not affect at all the special problem with which I was concerned which was this: What is the genetic difference between a race of rabbits which grows to an average size of 1400 grams and a race which grows to an average size of 3600 grams. I found that this genetic difference expresses itself in all parts of the body studied, such as the length of the skull, and the bones of the limbs, the ear-length, and the total adult weight (in health and normal flesh). Any one of these could be taken as an index of general body size, because each of them was closely correlated with every other one. Sumner does not dispute this. He says, "It may, of course, be replied that this, in itself, is evidence that the growth of the several members is conditioned by factors affecting the growth of the entire body. If by this is meant merely that the size attained by the various parts of the body is in a large measure dependent upon the size attained by the body as a whole, I should not think of disputing the proposition. The question at issue is whether, despite this pronounced tendency for all of the parts to grow in unison, there is not a considerable margin of independent variability in respect to the size attained by any one of them." I have tried to show that Sumner and I are really *not* "at issue" here. What I have been studying is general body size; what he is discussing is variation in parts independent of general body size. Neither he nor I can say to what extent such independent variation occurs. I get no evidence at all in rabbits of the occurrence of anything more than *somatic* variation in the size of the parts of the body indepen-

dently of general body size. He suggests that, if I were to *keep the size of the body constant* by the method of partial coefficients, I would probably find evidence of independent variation, some of which at least would have a genetic basis, that is, be inherited. I doubt whether this method is applicable to the case. How are we going to keep the body size constant in dealing with two races of rabbits which differ chiefly in body size and one of which is three times as big as the other? I doubt whether this statistical method is applicable to the biological facts. We cannot increase the actual size of a pure Polish rabbit from the usual 1400 grams weight and 84 mm. ear-length to the 3600 grams weight and 145 mm. ear-length of a Flemish Giant. By no process of mathematical reasoning can we say how long ears a Polish rabbit *would* have if we could make him grow to the size of a Flemish Giant. We can ascertain how long ears a Flemish Giant rabbit *has* when he is no larger than a Polish rabbit. His ears are actually *much longer* than those of a Polish rabbit, for the growth rate of the ears in early life is much more rapid than that of the body in general and the ears cease to elongate long before the body is full grown. If now we were to "make body size constant" by comparing only Flemish and Polish rabbits of the same actual body size, or by treating our data by any mathematical procedure which would lead to a similar result, we should get the idea that ear-length is extremely variable in relation to body size, which is not the case. For by crosses we have been able to produce a large series of rabbits intermediate in size between Polish and Flemish, inheriting their size from these races, and seriation of the data, as in Tables 12–16,[2] shows that, confining our attention to adult rabbits, there occurs with increase in weight, skull-length, or length of the leg-bones, *pari passu* an increase in ear-length, so that it is perfectly evident without any mathematical treatment whatever that there is a high degree of interdependence between general size and ear-length. Treating the data statistically, the coefficient of correlation is found to be exactly $0.836 \pm .011$ for both ear-length, skull-length, and ear-length weight. That the high degree of correlation shown is due to genetic agencies (heredity) is demonstrated by the breeding records which are incorporated in my paper.[2] That there are no discoverable special factors for ear-length or skull-length independent in action from the factors for general size is shown by the fact that in F_2 there are found no rabbits with ears or skulls either *especially* long or *especially* short in relation to general body size, as also by the fact that in F_2 the correlation between pairs of characters is substantially the same as in F_1. Sumner objects that it is not legitimate for me to include in one correlation table individuals from two distinct races and their cross-bred descendants. But I have in one case (femur-humerus) treated each group separately and found that it makes no great difference. Combining all the races, P, F_1 and F_2, in one correlation table, as I have regularly done for all

characters, we get a correlation coefficient for the femur-humerus pair of characters of 0.906. The F_1 group considered by itself gives a correlation coefficient of 0.888, and the F_2 group gives 0.878, values remarkably close together. Sumner's mixing together two different subspecies of Peromyscus so as to get a population showing a negative correlation whereas each race by itself gave a positive correlation, is not a parallel case. My mixing of two races of rabbits is not a mechanical but a genetic mixing.

The fact that I have found no evidence of the existence of special size factors in rabbits does not of course prove that such factors do not exist elsewhere. I have already pointed out that Mendelian mutations affecting the size of special parts are well known. I may add another example. The brachydactyl mutation in man is one which affects the length of bones both in the vertebral column and in the appendages. Most mutations of this sort have only a small and indirect influence on total size; but it is conceivable that a mutation might occur, let us say in rabbits, which would arrest growth sooner or prolong growth longer than is usual, resulting in the production of dwarf or giant races with different proportions of parts. In reality, however, we get no evidence that dwarf or giant races have originated in this way, since the inheritance of general size differences is regularly blending. More probably racial changes in size and proportions are gradual and slow, their evolutionary progress being recorded in geographic groups or sub-species, such as Sumner has been studying. He is to be commended for having undertaken a study of them, involving as it does great difficulties but not insurmountable ones.

I quite agree with Sumner's conclusion that on the basis of comparative anatomy we must suppose that changes in proportions of parts occur quite apart from such changes as result from a general increase or decrease in total size. If they occur by mutation it must be by such minute steps as to be practically imperceptible, since the breeding evidence available indicates a blending inheritance which only multiple factors can account for on a Mendelian basis. I share Sumner's doubt as to "whether in reality the phenomena of size inheritance are satisfactorily accounted for in Mendelian terms," and I should be glad to employ other terms, could I find any equally apt to express the facts and equally apt not to be misunderstood. Having conceded that *whole chromosomes* are concerned in heredity, as Sumner does, it will not be a difficult step to concede that *part-chromosomes* are concerned in it, and that these parts may be very minute and very numerous, which is the substance, as I understand it, of the multiple factor theory as applied to blending inheritance.

[1] Sumner, these PROCEEDINGS, Nov. 1923.

[2] Castle, *Carnegie Institution Publ.*, 1922 (320).

DISCOVERIES DURING THE SEASON OF 1923 BY THE THIRD ASIATIC EXPEDITION IN MONGOLIA

BY HENRY FAIRFIELD OSBORN

AMERICAN MUSEUM OF NATURAL HISTORY

Read before the Academy, November 14, 1923

To eastern Mongolia under the guidance of Roy Chapman Andrews, leader of the Third Asiatic Expedition, which has now been in this field for three seasons, the writer made a rapid journey examining personally the fossil beds surrounding Iren Dabasu, at the lowest point of the eastern end of the Gobi Desert on the Urga trail. The three formations exposed here are:

Houldjin beds, Baluchitherium Zone, Upper Oligocene

Irdin Manha beds, Protitanotherium Zone, Upper Eocene

Iren Dabasu beds, Middle Cretaceous.

The Houldjin Formation is of historic interest as including the spot where the Russian explorer, Obruchev, found a single rhinoceros tooth, probably belonging to *Baluchitherium;* also because the first fossil by the Third Asiatic Expedition was found here. This eastern exposure of the Baluchitherium zone has very rich and fragmentary remains. The finest specimens of Baluchitherium came from the far western exposure of Hsanda Gol.

The Irdin Manha is extraordinarily rich in Titanotheres and other mammals of exactly the same geologic age as the Uinta C beds of Utah, at the very close of Eocene time. Superb collections were obtained from these beds in the type locality, also ninety-eight miles west at Ula Usa.

The Iren Dabasu ("valley of the salt lake") beds yielded rich littoral fauna, of iguanodonts, of dinosaurs, of carnivorous dinosaurs and of toothless herbivorous dinosaurs known as *Ornithomimus* or *Struthiomimus*, which will enable us to determine precisely the geologic age of these beds, probably lower levels of Upper Cretaceous.

The Third Asiatic party this season consisted of Leader Roy Chapman Andrews, Frederick E. Morris, geologist and topographer, and Walter Granger, vertebrate palaeontologist, with three field assistants, Messrs. Kaisen, Olsen and Johnson, all highly experienced in Montana and Wyoming exploration. This strong party returned to the locality in eastern Mongolia where a single small type specimen of *Protoceratops andrewsi* was discovered during the season of 1922 and assigned to the Lower Cretaceous. Here the most remarkable discoveries of the year were made, namely, dinosaur beds of unparalleled richness, probably of Lower Cretaceous age and of Aeolian origin. The dominant element in the fauna is Protoceratops, which is found in all stages of development from the

young still contained within the egg before hatching to the fully adult. This proves to be a veritable ancestor of the Ceratopsia, with a well-developed neck frill, with rudiments of horns above the eye and also beneath the nasals. Seventy-two skulls and twelve more or less complete skeletons of this remarkable animal were unearthed and transported by camel caravan 800 miles across the desert to Kalgan, where they have recently arrived, as announced by cable. With Protoceratops were found many other kinds of reptiles, affinities of which have thus far not been determined.

To sum up the season 1923, out of the thirteen fossil-bearing horizons discovered in 1922, seven were extensively explored; five of these yielded very rich fossil results, which in time will enable us to determine precisely their geologic age. Mongolia is proved to have been highly fertile, a richly inhabited country from the close of Tertiary time, an evolution center—possibly the chief evolution center of land reptiles during the Age of Reptiles and a very important evolution center of the land mammals during the Age of Mammals.

SPECTRA OF METEOR TRAINS

BY C. C. TROWBRIDGE (*Deceased*)

DEPARTMENT OF PHYSICS, COLUMBIA UNIVERSITY

Communicated, October 20, 1923

NOTE: The material for this paper was completed by Professor Trowbridge shortly before his death in June 1918. The paper was actually written by Miss Mabel Weil, who worked with Professor Trowbridge and who had learned from him the form in which he had intended to publish this research. This work was carried out and is published with the aid of grants from the J. LAWRENCE SMITH FUND of the NATIONAL ACADEMY OF SCIENCES.

1. *Historical Summary.*—The origin of meteor trains, or the persistent phosphorescent streaks left by large meteors, has been one of the enigmas of astronomy. Many of the streaks observed persist for several minutes, sometimes for an hour or more, before gradually fading from view. The meteor trains observed at night are self-luminous, for they appear in a part of the sky where there is no possibility of reflected sunlight and they often have discontinuous bright line spectra. The present investigator has, in previous papers, expressed the view that meteor trains are probably due to the phosphorescent afterglow in a gas at extremely low pressure. An investigation regarding the spectra of these bodies would prove of

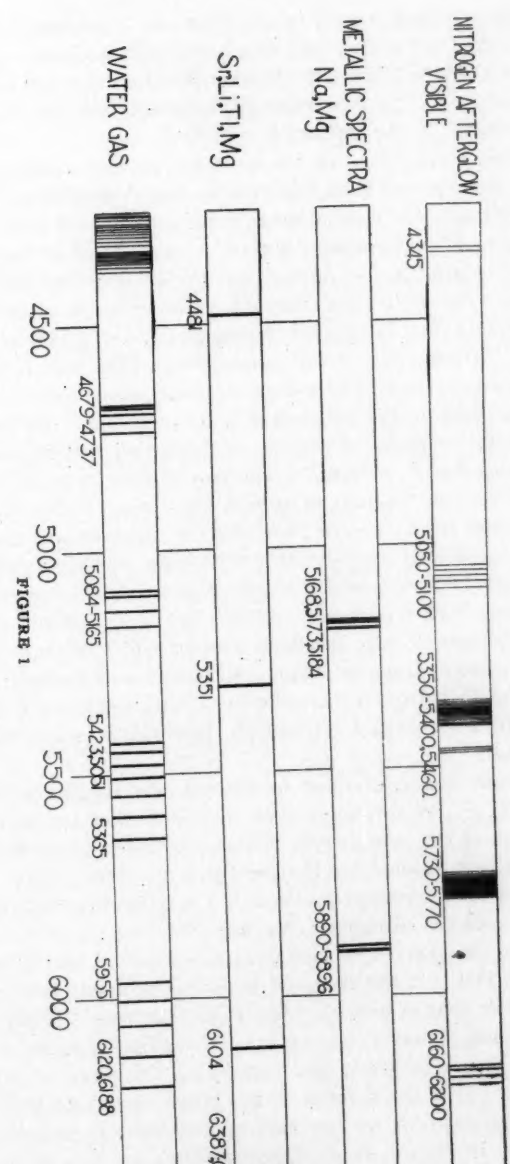
NITROGEN AFTERGLOW
VISIBLE
4345
5050-5100
5350-5400,5460
5730-5770
6160-6200
METALLIC SPECTRA
Na,Mg
5168,5173,5184
5890-5896
Sr,Li,Tl,Mg
4481
5351
6104
6387,6409
WATER GAS
4679-4737
5084-5165
5423,5051
5365
5955
6120,6188
4500
5000
5500
6000

FIGURE 1

considerable assistance in deciding this question. Comparatively little work has been carried out in this field of research, but the evidence, so far as it is available, tends to show that the identifications of spectrum lines made by most observers of meteor trains are incorrect, and tend to confirm the theory advanced by the present investigator.

The earliest observations on the spectra of meteor trains were made by Alexander Herschel and John Browning in 1866. Neither of these investigators could have done very accurate work, as they both used direct vision spectroscopes of low dispersion without a slit. Most of the spectra observed were continuous or diffuse, but the observations included a few line spectra. Secchi, in 1868, observed a discontinuous meteor train spectrum containing lines in the red, yellow and green, which he was unable to identify. During the period from 1873 to 1879, von Konkoly, using a Browning direct-vision spectroscope, examined many meteor train spectra and observed lines in the red, yellow, green and blue. He identified one of these spectra with that of coal gas, and believed that in another he had observed lines due to sodium, magnesium, copper, iron, and one of the elements strontium, lithium or potassium. Most of his identifications were made from memory some time after his observations, and hence can hardly be considered trustworthy. Pickering photographed a meteor train spectrum in 1897 in which he considered the presence of ultra-violet hydrogen lines highly probable. Blajko, in 1904, believed that he had identified photographically the lines of hydrogen, potassium and magnesium in one meteor train spectrum, and helium and thallium in another. The discrepancies between observed wave-lengths and those of the elements mentioned by him, seem great enough, however, to make identification very doubtful.

The spectrum of the afterglow in nitrogen was studied by E. P. Lewis in 1900, by C. C. Trowbridge in 1906, and by Fowler and Strutt in 1911. The spectrum of the afterglow in helium was observed for the first time and qualitatively studied by Bergen Davis in 1905. Three of the six lines of this spectrum occupy positions not very far from lines in the spectrum phosphorescent nitrogen. The afterglow spectra of other elements have received very little attention from investigators, and little is known excepting the fact that the afterglow in oxygen probably has a continuous spectrum, while that in hydrogen has a line spectrum. Many facts have been determined, however, concerning the conditions under which phosphorescence occurs in gases, and concerning the physical properties of the afterglow. It is the purpose of the present paper to prove that the spectrum lines observed by the various investigators mentioned, might just as readily be lines in an afterglow spectrum as those of the elements mentioned. An impartial consideration of the details of the work of these investigators as well as of the spectrum and properties of the afterglow and

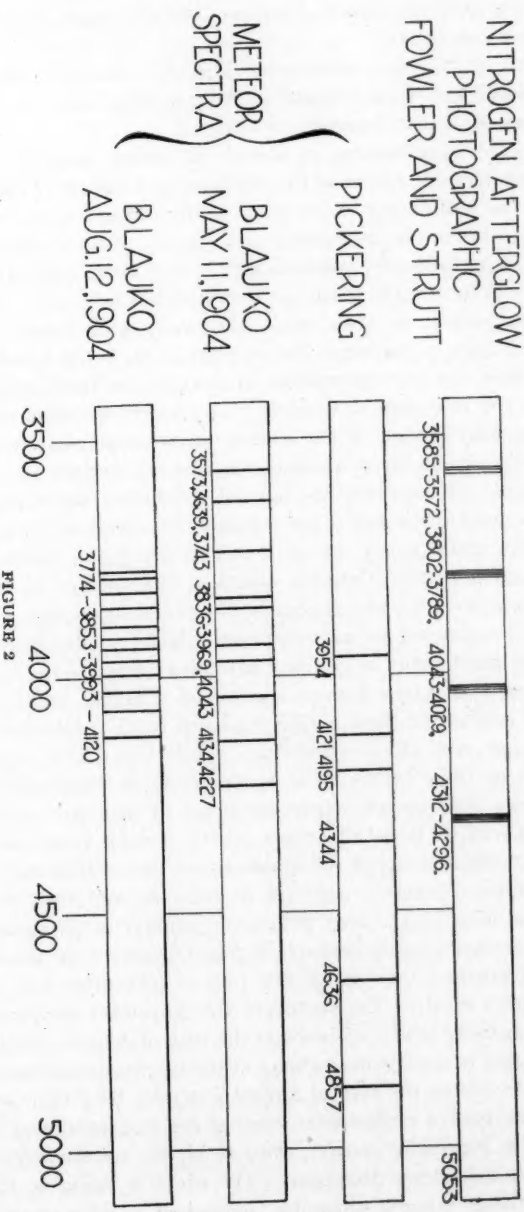
NITROGEN AFTERGLOW
PHOTOGRAPHIC
FOWLER AND STRUTT
METEOR SPECTRA
PICKERING
BLAJKO MAY 11,1904
BLAJKO AUG.12,1904
3585-3572, 3802-3789, 4043-4029, 4312-4296.
5053
3954
4121 4195 4344
4636
4857
3573,3639,3743 3836-3969, 4043 4134,4227
3774 - 3853-3993 - 4120
3500
4000
4500
5000

FIGURE 2

the probable physical nature of meteors and the upper atmosphere, will lead to this conclusion.

2. *Work of Herschel.*—Alexander Herschel observed the spectra of many meteors and meteor trains in August 1866, using a direct vision spectroscope which he describes as follows:

"The meteor spectroscope, as already described, presents to the view a pretty considerable extent of the star-spangled surface of the sky. The spectra of the well-known 'seven stars' of Ursa Major may, for example, be seen together in the instrument at a glance. Each bright star is converted into a line of highly coloured light, nearly three-quarters of a degree in length; and horizontal, when the instrument is held in its natural position. Fifth magnitude stars are obliterated, and fourth magnitude stars appear only as a greyish line of light of no decided tint or colour. Prismatic hues are first perceptible in stars of the third magnitude and upwards. . . the stars appear to occupy very nearly the same relative positions in the field of view of the meteor spectroscope as that which they occupy in the sky to the unassisted eye. In this manner the instrument is well adapted for analyzing the light of any bright object presenting itself for a moment in the part of the sky under examination, as, for example, the light of a shooting-star, or of its bright, fast-fading luminous train."

During August 1866 Herschel observed the spectra of twenty-one meteors, most of which had trains. "The result shows that some of the streaks were composed of monochromatic light." The observations on meteor train spectra may be grouped into three classes, in the first of which the spectrum is a yellow line, in the second a diffuse band including a yellow line, and in the third a diffuse greyish band. Herschel identified the yellow line with the sodium line. He further added regarding the constitution of these bodies:—"If the problem of chemically analysing the substance of luminous meteors by means of their light spectra is not yet fairly solved, it is at all events pretty certain, from the following observations, that the metal sodium produces the most enduring light of the much-admired trains of the August meteors, and that at least one other mineral substance (either potassium, sulphur or phosphorus) lends its aid, but in a much less remarkable degree, to produce the same luminous trains. Observations renewed on the 13th of November will, doubtless, show a different result. The streaks of the November meteors are quite as enduring as those left by meteors on the 10th of August; but their color is white, verging to blue, while a glance at the brightest and most enduring meteor streaks left on the 10th of August generally show their yellow cast of color. The first or rudimentary color of the August meteor streaks is, like that of the November streaks, white or bluish, and some few continue of this colour until they disappear. The effect is owing to the ignited vapor of the other mineral substance (potassium, sulphur or phosphorus)

LEWIS
1900
TROWBRIDGE
1906
FOWLER
AND
STRUTT
1911
4345
5005
5410
5740,5780,5845
6175,6245,6320
4500
5000
5500
6000
5051-5054
5050-5100
5350-5400
5460±5
5730-5770
6160-6200
5372-5407
5442
5755-5804
5855
6185-6253
6323

FIGURE 3

to which allusion was just made, playing a principal part in the production of the streaks. Although glowing for a much shorter time, and with less intensity than the vapour of the metal sodium, it nevertheless in some instances forms the entire light of the meteor streaks seen on the 10th of August. White, bluish, or 'phosphorescent' streaks are most prevalent among the November meteors, and the light of this element, whatever it be, by which this bluish kind of phosphorescence of the streaks is produced, will probably appear more highly developed and its spectrum will be more easily identified in the streaks of the meteors on the 13th of November, than it could be (from the brightness of the sodium line) in the enduring streaks of the meteors of the 10th of August last. It extends between the red and the blue; and is that portion of the visible train spectrum which is called 'diffuse' in the following observations." In another part of the same paper Herschel says: "No lines of nitrogen, or of any other known gas, accompany the sodium flame in the train spectra of the August meteors, but, on the other hand, the great intensity of the sodium streaks makes it almost certain that the meteors contain this element as a part and parcel of their chemical composition."

Herschel gives no reasons as to why he considers the presence of phosphorus, sulphur or potassium so highly probable, as he observed no lines of these elements in the spectra, but, in addition to the yellow line, nothing but a diffuse band. It is interesting to note that Herschel seemed quite certain that he was observing a metallic spectrum and not that of a gas. His observations, however, were subject to considerable error due to the imperfections of the spectroscope used. By comparing the first two spectra in figure 1 with the first and fifth of figure 4, it can readily be seen that it is just as probable that the yellow line observed by Herschel was the yellow line of the afterglow spectrum of nitrogen or helium as the yellow sodium lines.

3. *Work of Browning.*—Browning like Herschel observed many diffuse and continuous spectra. His descriptions, as a rule, are not clear, and in many cases it is uncertain whether he is describing the spectra of nuclei or trains. Browning mentions continuous spectra in which yellow is strongly predominant, spectra containing a bright orange-yellow line, and others containing a green line of nearly the same color as that of thallium. The direct-vision spectroscope used by Browning was criticized as follows at a meeting of the Royal Astronomical Society:

The President—"The light from the trains, when green, appeared continuous in the prisms; and the blue trains gave a line of lavender, with occasional traces of a continuous spectrum—Then we may take it that without a slit the accuracy of the spectroscope does not exceed that of a blow-pipe examination of a substance."

Mr. Browning—"Not exactly: in the spectra of trains, there are distinct lines."

Just what Browning meant by a "lavender" line, it is extremely difficult to state. It seems most probable that he observed a diffuse or continuous spectrum.

4. Work of Secchi on Meteor Train Spectra.—In 1868, Secchi examined many meteor trains with the spectrometer and believed that he had observed the lines of sodium and magnesium. In particular, he described

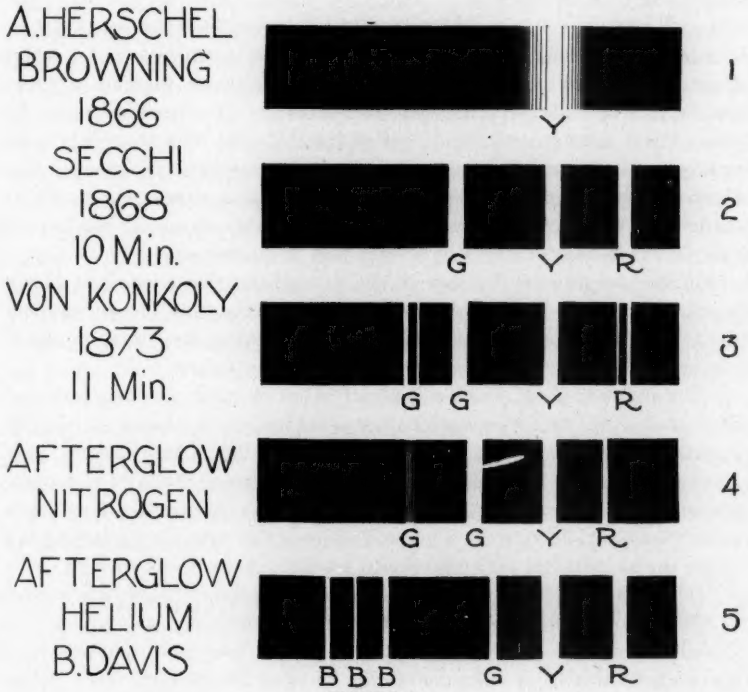

FIGURE 4

one spectrum which seemed of unusual interest. This was discontinuous and contained bright lines in the red, yellow and green, as indicated in figure 4. The observation must have been unusually good, as the train lasted for ten minutes and was observed during all of that time. Secchi made no attempt to identify these lines. His own description of his observation as published in his article, "Le stelle cadenti del 14N ovembre, 1868," is quoted below:

"Merita attenzione, e deve esser segnalata una superba stella che si accese alle 4 or 51 circa accanto a Regolo, lasciando una stretta e corta

nube, in forma di arco lungo un grado al pus, ma di grande vivacita, sicche pote analizzarsene la luce comodamente allo spettrometro, e si ebbe uno spettro tutto discontinuo di rihge vivissime nel rosso, nel verde e nel giallo. Quest arco si andò piano dilatando e allargando fino a formare un gran cerclio opposto alla Falce del Leone. Non vi par dubbio che queste apparenze fossero quelle di una traiet toria spirale veduta secondo il suo asse. La luce duro per ben 10 minuti."

The similarity of this spectrum to the spectrum of the afterglow can readily be seen in figure 4.

5. *Observations of von Konkoly.*—The work of Nicolaus von Konkoly is interesting because of the unusually favorable conditions under which it was carried out, and of the inexact and unscientific manner in which conclusions were drawn from the observations. Von Konkoly made his observations between 1873 and 1879 at the O'Gyalla Astrophysical Observatory in Hungary. He used a Browning direct-vision spectroscope and compared the spectra observed with Geissler tube spectra. He may have made some of his comparisons at the time of observation, but it is certain that he made most of them at a later time from memory.

The meteor train of October 13, 1873, is of particular interest, as von Konkoly was able to observe the phenomenon for fully eleven minutes. A literal translation of the portion of his article in the "Astronomische Nachrichten" describing his observations, is as follows:

"The streak of light, which was about 15′ or 20′ wide, was bright enough so that it could be investigated spectroscopically. I wished to examine it with a Browning meteor spectroscope, with which I immediately identified sodium and magnesium, but also saw several other bright bands whose presence I could not account for. As I considered the object sufficiently brilliant, I screwed a 5-prism Browning direct-vision stellar spectroscope to the refractor and carried out further observations with this.

"In this way, I saw, besides sodium and magnesium, four other bands of light, two in the red and two in the green. I compared these with several Geissler tubes and finally found that these four lines fully agreed with a spectrum shown by a tube containing rarefied illuminating gas. When I completed my spectroscopic investigations, it was 9 hr. 52 min. Therefore I had fully eleven minutes in which to investigate the spectrum."

Von Konkoly describes in full another spectrum which he observed for thirty seconds. In this, also, he observed the yellow and green lines which he attributed to sodium and magnesium, respectively, and also saw a number of other lines in the green and blue, which he thought were due to heavy metals, in particular to iron. On a later day, he made an alcoholic solution of sodium, magnesium, iron and a little copper, then he moistened a ball of cotton with this solution, had it tossed in the air in a dark place and observed the spectrum. As this appeared similar to the spectra he had ob-

served on a previous day, he concluded that he had identified the elements sodium, magnesium, iron and copper in the meteor train spectrum.

Von Konkoly described certain other spectra in his articles, which are probably those of nuclei and not trains, although he does not make the distinction at all clear excepting in the two cases described above. He mentions certain spectra containing the red lines of lithium, strontium and potassium, also the sodium and magnesium lines previously described, but these are also probably spectra of nuclei. In the case of the identification of coal gas, he was not absolutely certain as regards the gas contained in the Geissler tube, but thought that it was coal gas or ethylene C_2H_4, "Leuchtgas womit die Strassen beleuchtet sind." Then again, he states that the entire observation lasted for eleven minutes, but it seems hardly possible that he could have made any really reliable comparisons in so short a time. If, on the other hand, he compared his Geissler tube spectra at a later time with the meteor train spectrum as he remembered it, his conclusions are equally untrustworthy. It does not seem possible from what is known of the nature of meteors or of their trains that the latter could be composed of coal gas. It must also be noted that von Konkoly observed two red bands and two green ones which he attributed to coal gas. The other green band and the yellow one, he considered to be nearer to positions of magnesium and sodium, respectively. According to Eder and Valenta, the visible spectrum of coal gas at low dispersion consists of a red band, a yellow, a green, a blue, and two violet bands. These bands contain several well-marked flutings, which, in the case of the yellow, green and blue bands, appear with high dispersion as numerous fine lines. It is extremely probable that von Konkoly identified his spectrum with coal gas simply because this is a well-known substance. From table I it can be seen that there are certain points of resemblance in the

TABLE I

BAND SPECTRA OF COAL GAS AND OF THE NITROGEN AFTERGLOW

COAL GAS		AFTERGLOW
*4190–4314	Violet	
*4324–4380	Violet	4345
*4679–4737	Blue	
5084–5165	Green	5050–5100
		5350–5400
		5460
5471–5635	Yellow	
		5730–5770
5955–6188	Red	6160–6200

Bands marked * were not observed by von Konkoly.

general appearance of the spectra of coal gas and of the nitrogen afterglow, so that in very rough qualitative work, one might be mistaken for the other.

6. Photographic Study of the Spectra of Meteor Trains.—Doubtless the

most careful piece of work as yet completed on the spectra of meteor trains is the observation made at the Arequipa Observatory on June 18, 1897, and described thus by Pickering in Circular No. 20 of the Harvard College Observatory:

"At about 11 P.M. on June 18, 1897, however, when the eight-inch Bache telescope at Arequipa was directed towards the constellation Telescopium, a bright meteor appeared in right ascension 18 hr. 19 min. declination $-47°$-10′, and passed out of the field at right ascension 18 hr. 29 min. declination $-50°30'$. The spectrum consists of six bright lines whose intensity varies in different portions of the photograph, thereby showing that the light of the meteor changed as its image passed across the plate. The approximate wave-lengths of these lines are 3954, 4121, 4195, 4344, 4636 and 4857, and their intensities are estimated at 40, 100, 2, 13, 10 and 10, respectively. The first, second, fourth and sixth of these lines are probably identical with the hydrogen lines, Hϵ, Hδ, Hγ, and Hβ whose wave-lengths are 3970, 4101, 4341, and 4862. The fifth line is probably identical with the band at wave-length 4633, present in spectra of stars of the fifth type and forming the distinctive feature of the class of these stars. The third line, which is barely visible, is perhaps identical with the band at wave-length 4300, contained in these stars."

This photograph was obtained on one of many thousand plates covering the entire sky, taken at the Cambridge and Arequipa Stations of the Harvard College Observatory, and examined by Mrs. Fleming. The photographs were made by placing a large prism over the object glass of a telescope and thus obtaining the spectra of all the bright stars in the field of view. The number of sharply defined spectra on one plate was increased by using a very large portrait lens in place of the object glass, thus increasing the field of view from two degrees square to ten degrees square. Pickering assigns some meteor-train spectrum lines to hydrogen in spite of the fact that the differences are sometimes as much as 20 A. This is not sufficiently precise to enable one to be certain that these lines do not belong to some far less familiar spectrum. On the basis of careful measurements and comparisons with accurately determined wave-lengths in known spectra, Pickering was certainly justified in drawing the conclusions described in his circular. This, however, does not preclude the possibility that the wave-lengths measured may agree much better with those of some spectrum which he did not have in mind when he made his comparisons.

Another photographic determination was published by Blajko in the *Astrophysical Journal*, 1907. Blajko, like Pickering, photographed only the violet end of the spectrum. There is a possibility that the spectra studied by him may be those of meteor nuclei rather than of trains, as he does not make this point perfectly clear. Blajko used a prismatic camera

for his observations, and measured his spectra on a Troughton measuring machine, comparing his wave-lengths with those of the hydrogen spectrum.

The first meteor observed was that of Mayll, 1904. He observed ten lines, all in the violet and ultra-violet region. He corrected his wave-lengths for the prismatic effect of the camera, the correction amounting to from 5 to 9 A. He made the following identifications:

CORRECTED WAVE-LENGTHS		
		3829.5
3835.5	corresponds to Mg lines	3832.5
		3838.4
4042.5	corresponds to K lines	4044.3
		4047.5

3933.5 and 3968.7 correspond to H and K calcium lines of the solar spectrum.

The second meteor spectrum was photographed August 12, 1904. Here the corrections reach from ten to thirteen units. In this case thirteen lines were observed, and the wave-lengths did not agree, for the most part, with those of the other spectrum. Blajko identified some of the lines which he photographed with those of helium as follows:

BLAJKO	HELIUM LINES (Runge and Paschen)
3807.4	3819.8
3879.1	3888.8
3953.7	3964.9
4016.8	4026.3
4110.3	4121.0

Blajko also states that the wave-length 3761.1 corresponds to the thallium line at 3775.9, and bases his identification on the green color of the meteor. Blajko identified the lines which he observed with those differing from them in wave-length by as much as twelve units after making his corrections.

It is unfortunate that there are not other observations on meteor train spectra in which the wave-lengths have been determined. The present investigator has three photographs of a spectrum in the files of material on meteor trains, but the lines have not been identified. They are all copies, two of them much enlarged, of a meteor train spectrum photographed May 18, 1909, at Arequipa and presented to the investigator by Prof. Pickering, director of the Harvard College Observatory. This spectrum was studied with much interest by Sir William Ramsay, who was unable to identify the lines. With the exception of this spectrum and those described above, there seems to be only one continuous spectrum which was photographed at the Harvard College Observatory. So, unfortunately, the work in this field is extremely limited.

A complete list of all lines measured in meteor train spectra is given in table II, together with identifications made.

TABLE II

LINES IN METEOR TRAIN SPECTRA

Pickering Meteor of June 18, 1897

WAVE-LENGTH	IDENTIFICATION	ELEMENT
3954	Hydrogen	$H\epsilon$
4121	Hydrogen	$H\delta$
4195	4200	Stars of 5th type
4344	Hydrogen	$H\gamma$
4636	4633	Stars of 5th type
4857	Hydrogen	$H\beta$

Blajko Meteor of May 11, 1904, identification element			Blajko Meteor of Aug. 12, 1904		
3573.0	....		3774.1	3775.9	Tl
3638.5	....		3790.3	....	
3742.8	....		3802.9	....	
	3829.5		3820.1	3819.8	He
3835.5	3832.5	Mg	3852.5	....	
	3838.4		3890.6	3888.8	He
3857.1	....	Ca	3915.1	....	
3933.5	Hand K lines	Ca	3938.5	....	
3968.7	....		3964.3	3964.9	He
4042.5	4044.3		3992.6	....	
	4047.5	K	4027.0	4026.3	He
4134.4	....		4066.5	....	
4227.2	....		4120.3	4121.0	He

7. *Spectrum of the Afterglow.*—Meteor trains resemble in several respects the phosphorescent afterglow of nitrogen and other gases. The properties of the afterglow of various elements have been studied, but as regards spectroscopic measurements, attention has been given principally to nitrogen. The afterglow of nitrogen was studied spectroscopically by T. P. Lewis in 1900, C. C. Trowbridge in 1906 and by Strutt and Fowler in 1911.

In his work on the afterglow, Lewis used some vacuum tubes with internal electrodes, others with external electrodes and a Tesla coil. With pure nitrogen very free from oxygen, he could obtain no afterglow with a current from an induction coil until he placed a condenser and spark gap in the secondary circuit, when he obtained a chamois-yellow mist. Lewis observed in the spectrum of the afterglow four bands, the wavelengths of whose centers were approximately 5410, 5740, 5780 and 6240. A few weak lines in the red were also observable in a pocket spectroscope of small dispersion. In a spectroscope of high dispersion all the bands appeared weak and diffuse. In making the photographs, a bent tube with a quartz window gave an end-on view of the afterglow, and screened the direct discharge.

The present investigator made a set of measurements of the afterglow of the electrodeless discharge using a direct-vision spectroscope. The

visual spectrum was found to be composed of four prominent bands, and several faint ones, two of which appear to be lines rather than bands. The wave-lengths are given in the appended table:

TABLE III

4345 ± 5	Very faint band or line
5100 – 5050	Very faint band
5400 – 5350	Band, not as bright as 5460
5460 ± 5	Line
5770 – 5730	Band, more intense in middle
6200 – 6160	Band, due to presence of water vapor

The lines 4345 and 5460 correspond quite closely to the mercury lines 4340 and 5460 and probably are accounted for by the mercury in the pump. The lines and bands were found to disappear in the following order:

(5100–5050), (6200–6160), 4345, (5770–5730), 5460, (5400–5350)

Fowler and Strutt studied the afterglow spectrum by means of three quartz spectrographs: one with dispersion of 5.5 cm. from 2200 to 7000, the second of 11.5 cm. dispersion, and the third a Littrow spectrograph with a prism of light flint glass dispersion 16 cm. from 3800 to 7000. Comparison spectra of copper were used for wave-length determinations. They reached the conclusions that the afterglow contains some of the same bands as the nitrogen spectrum but that the intensities are different. "Apart from the modification of intensities, for which we can offer no explanation at present, this observation is in accordance with the view that nitrogen is partially dissociated in the condensed discharge, and that the afterglow is produced during the re-formation of ordinary nitrogen." The wave-lengths found by Fowler and Strutt are given in table IV.

TABLE IV

.....	4296.5	.....
.....	4312.3	5755.20
3572.9	5030.8	5804.28
3584.7	5053.6	5854.69
3789.3	.....	.....
3801.9	5372.78	6185.44
4028.7	5407.08	6252.81
4042.8	5442.25	6322.73

All of these spectra are shown in figure 3. The afterglow of oxygen has been found to have a continuous spectrum and that of hydrogen has been found by Hertz to have at least ten lines between the green and the violet.

The afterglow of helium was observed and qualitatively studied by Davis in 1905. A spectrum containing six lines, one red, one yellow, one green and three in the blue. The lines disappeared in the following order: three blue, green, red, yellow. Both red and yellow lines were quite persistent, while the others disappeared almost immediately.

In his previous publications on the subject, the present investigator has

pointed out certain strong resemblances between meteor trains and the afterglow. The rate of diffusion is of the same order in both cases. The afterglow persists for several minutes, sometimes as long as twenty minutes, and meteor trains sometimes persist for even a longer period. The afterglow can continue to exist even at liquid air temperatures, and it seems quite certain that at the high altitudes (50 or 60 miles) at which meteor trains occur, the temperatures are extremely low. Both afterglow and meteor trains appear to have bright line or band spectra. The afterglow appears only within certain limits of pressure, and it is quite probable that at a height of 50 or 60 miles similar atmospheric pressures occur. The agreement of the spectra in their general features seems merely to be further confirmatory evidence of the coincidence of these two phenomena. A glance at figure 4 will show in a general way the coincidence between the visual portions of spectra as observed by Browning, Herschel, Secchi and von Konkoly with the spectra of the afterglow of nitrogen and helium. The identifications with sodium, magnesium, etc., were probable made simply because these substances were common and well known. If all the streets of London at that time had been illuminated with the light of phosphorescent nitrogen, Browning and Herschel would undoubtedly have identified the yellow band which they observed with the yellow band of the nitrogen afterglow spectrum. The work of all the earlier investigators must be considered as purely qualitative. The photographic measurements by recent investigators, unfortunately, cover different portions of the violet and ultra-violet spectrum, and do not overlap to any great extent, so it is somewhat difficult to make comparisons with them. There are, however, a few striking agreements among the measurements made.

In the appended table, lines in meteor train spectra corresponding to those in the afterglow will be placed side by side.

TABLE V

AFTERGLOW	METEOR TRAIN	INVESTIGATOR
3572.9		
3584.7	3573	Blajko—first train
3789.3	3802.9	Blajko—second meteor
3101.9		
	4027.0	Blajko—second meteor
4028.7	4042.5	Blajko—first meteor
4042.8	4066.0	Blajko—second meteor

The preceding considerations would seem to make it far more reasonable to believe that meteor trains are gases in the phosphorescent state rather than the vapors of glowing metals. The present investigator outlines this idea in a previous paper as follows:

"The motion of the meteor through the atmosphere produces an exceedingly high temperature and may bring about chemical or physical

changes in the composition of the atmosphere in the track of the meteor; which on reverting to its original state gives out a phosphorescent glow, or the surrounding air may be highly ionized by the vaporizing meteor so that electrical discharges take place great enough to produce an afterglow like that following the electrodeless discharge."

Comparatively few attempts have been made to explain the nature of meteor trains. Most of the ideas expressed on this subject have been mere conjecture, as will be shown in a separate paper by Miss Weil on the theories of meteor trains. The agreement of the spectra of meteor trains with that of the afterglow is at least as good as the agreement with the spectra of certain other elements, and a comparison of the physical characteristics or the afterglow with those of meteor trains tends further toward the conclusion that the two phenomena are identical. As regards strictly quantitative work, only a beginning has been made, but a further study of the afterglow, of meteor trains and their relation to each other would greatly aid in the solution of problems connected with the height of the earth's atmosphere and its density, as well as the direction, and velocity of air currents in the upper atmosphere.

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ON TERTIARY X-RADIATION, ETC.

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In two notes[1] in these PROCEEDINGS for December 1923 we have described experiments on secondary X-radiation produced by primary X-rays from a tungsten target falling on secondary radiators consisting of various chemical elements. Measurements with an accurate spectrometer showed that within the limits of error (about .1%) the scattered radiation had the same wave-lengths as the primary X-rays, and that the fluorescent radiation had the same wave-lengths as the line spectra of the chemical elements obtained when they were used as targets. Further, evidence appeared of radiation in the case of a copper secondary radiation that may be interpreted as tertiary radiation due to the bombardment of the copper atoms by photo-electrons ejected from them by the primary rays. This tertiary radiation occupies a band in the spectrum that is broader than the lines in the primary radiation. The short wave-length limit of a tertiary radiation band may be calculated from the formula

$$h\nu = h\nu_1 + h\nu_2 \qquad (1)$$

where ν, ν_1 and ν_2 are the frequencies of the primary, the limit of the tertiary radiation and the critical absorption of the secondary radiator, respectively. From this we get the following equation for the difference between the short wave-length limit of the tertiary and the wave-length of the primary rays,

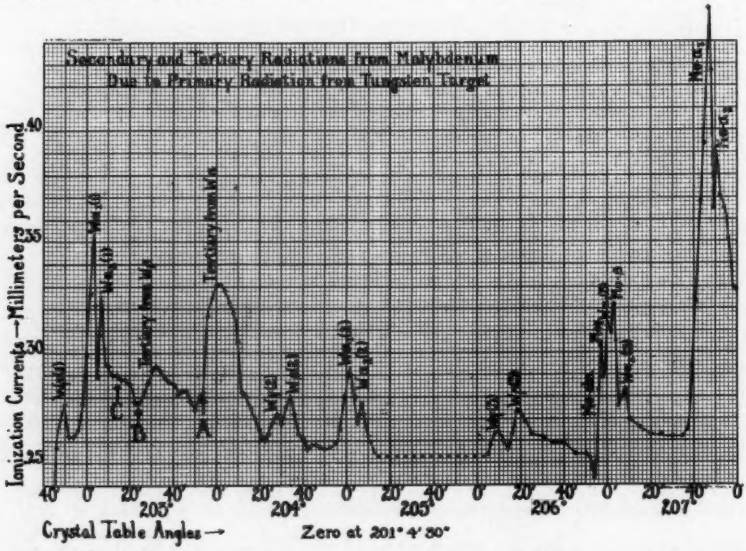
$$\lambda_1 - \lambda = \frac{\lambda^2}{\lambda_2 - \lambda} \qquad (2)$$

The formula indicates that the wave-length shift, $\lambda_1 - \lambda$, increases if the critical absorption wave-length, λ_2, of the secondary radiator decreases. The critical absorption wave-length of a chemical element grows smaller as the atomic number of the element increases. Hence, by taking a secondary radiator consisting of a chemical element of higher atomic number

than copper (N = 29), the wave-lengths of the tertiary radiation ought to shift towards a larger value.

The object of the present note is to describe experiments in which we have used molybdenum (N = 42) and silver (N = 49) as secondary radiators, the primary rays coming as before, from a tungsten target. Figure 1 in the first of the two notes above referred to illustrates the arrangement of the apparatus, the only change made being the substitution of plates of metallic molybdenum and silver as secondary radiators.

The curves in the figure of this note represent the spectrum of the radiation from a molybdenum plate due to primary radiation from the tungsten target of the X-ray tube. The curves give the ionization currents for



different settings of the crystal of the spectrometer on one side of the zero of the instrument only. The corresponding data for radiation on the other side of the zero were obtained and from them the exact zero for the setting of the crystal table was estimated to be at 201°4′30″. The three peaks at the extreme left of the curve in the figure correspond to the β, α_1, and α_2 lines in the K series of tungsten, as represented in the scattered radiation from the molybdenum plate. We wish to call attention especially to the fact that the resolving power of the spectrometer sufficed to separate the α_1 and α_2 lines from each other.

Further along in the spectrum, at an angle of about 204°50′, the second order spectrum of these same lines begins. Here the tungsten γ line, as well as the β, α_1 and α_2, appear separately. Further along still, the third

order spectrum of the γ and β lines appears at angles of about 206°10′ and 206°20′, respectively. The peaks corresponding to the third order reflections of the α_1 and α_2 lines fall close to and are intermingled with the γ and the β lines of the fluorescent K series spectrum of the molybdenum plate. All four of these lines appear separated from each other and the fact that they lie so close together furnishes an excellent means of comparing the wave-lengths of the molybdenum fluorescent spectrum with those of the scattered radiation corresponding with the tungsten spectrum. Much of the secondary radiation is produced within the substance of the molybdenum plate and before these rays reach the spectrometer they must pass through some molybdenum. The critical absorption of the molybdenum, the wave-length of which lies very close to the limit of the molybdenum K series spectrum, is shown on the curve as an abrupt rise at 206°56′. The two peaks at the extreme right of the curve represent the α_1 and α_2 lines in the fluorescent K series of molybdenum.

TABLE I

SECONDARY AND TERTIARY RADIATION FROM MOLYBDENUM DUE TO PRIMARY RADIATION FROM TUNGSTEN

Wave-Lengths Are Given in Ångströms. Zero of Crystal Table Angle = 201°4′30″

CRYSTAL TABLE READING	ANGLE, θ, OF INCIDENCE	ORDER, n, OF SPECTRUM	WAVE-LENGTH $n\lambda = 2d\sin\theta$	WAVE-LENGTH KNOWN OR CALCULATED	CHARACTER OF RADIATION
202°49′30″	1°45′00″	1	.1849	.1842	Tungsten Kβ
203° 2′55″	1°58′25″	1	.2086	.2086	Tungsten Kα_1
203° 6′40″	2° 2′10″	1	.2148	.2134	Tungsten Kα_2
203°10′22″	.....	1	...	...	Tertiary from L Electrons
203°24′00′	2°19′30″	1	.260	.262	Tertiary f. Wβ
203°54′00″	2°49′30″	1	.311	.315	Tertiary f. Wα
204°28′00″	3°23′30″	2	.1792	.1790	Tungsten Kγ
204°33′50″	3°29′20″	2	.1843	.1842	Tungsten Kβ
205° 1′30″	3°57′00″	2	.2086	.2086	Tungsten Kα_1
205° 7′00″	4° 2′30″	2	.2135	.2134	Tungsten Kα_2
206° 9′20″	5° 4′50″	3	.1789	.1790	Tungsten Kγ
206°19′00″	5°14′30″	3	.1844	.1842	Tungsten Kβ
206°55′50″	5°51′20″	1	.6180	.6184	Mo. Absorption
206°57′00″	5°52′30″	1	.6199	.6197	Molybdenum Kγ
207°00′15″	5°55′45″	3	.2085	.2086	Tungsten Kα_1
207° 3′00″	5°58′30″	1	.6307	.6311	Molybdenum Kβ
207° 8′45″	6° 4′15″	3	.2135	.2134	Tungsten Kα_2
207°47′10″	6°42′40″	1	.7078	.7078	Molybdenum Kα_1
207°50′00″	6°45′30″	1	.7126	.7121	Molybdenum Kα_2

As indicated in table I, the wave-lengths of the X-rays corresponding to these various peaks agree very closely with the measured and recorded wave-lengths of the K series lines of tungsten and of molybdenum, so that the data furnish additional evidence of the general correctness of the theory that scattered radiation has, essentially, the same wave-lengths

as the primary and that the wave-lengths of the fluorescent lines produced in a chemical element by primary X-rays are the same as those produced by the bombardment of electrons.

In addition to the peaks corresponding to the scattered and fluorescent radiation, several humps appear on the curve at A, B and C, respectively. These humps correspond to wave-lengths that agree very well with the wave-lengths of the tertiary radiation due to the bombardment of the atoms by the electrons emitted by the primary radiation. If we substitute in equation (2) for λ the wave-length of the α_1 line in the K series of tungsten, namely .2086, and for λ_2 the critical absorption wave-length in the K series of molybdenum, namely .6184, we find that the tertiary radiation due to the photo-electrons emitted by the tungsten α lines should have a short wave-length limit, equal to .315. Calculating from this the angle at which the corresponding rise in the curve should begin, we find that it ought to start at the point marked A in the figure. The hump beginning at this point, therefore, and lying to the right of it, represents tertiary radiation due to the electrons from the K level in molybdenum atoms that are ejected by the X-rays belonging to the primary α lines in the K series of tungsten. A similar calculation for the electrons emitted from the K levels of molybdenum by the tungsten β line rays shows that the hump corresponding to the tertiary radiation in this case should begin at the point marked B in the diagram. According to the above calculations, the point marked A should lie 1° to the right of the peak representing the α_1 line of tungsten, and the point marked B should lie 30′ to the right of the same peak. It must be remembered, however, that in estimating the positions of the points A and B, a certain correction has to be made for the angular breadth of the incident beam of X-rays. This correction amounts to half the breadth of a peak corresponding to a single line. What amounts to the same thing, however, is that the point A should lie 1° to the right of the point on the curve where the tungsten α_1 peak begins to rise, and the point marked B should lie 30′ to the right of the same point. Since the short wave-length limits of the two humps marked A and B actually fall so close to their theoretical positions, there can be no doubt but that the humps represent the tertiary radiation due to the electrons ejected from the K levels of the molybdenum atoms by the primary α and β radiation of tungsten.

There is evidence at the point C on the curve of some excess radiation. This may be interpreted as tertiary radiation due to the ejection of electrons from the L levels of the molybdenum atoms by the primary Kα X-rays. Calculation from formula (2) shows that the short wave-length limit of the tertiary radiation in this case should lie only about 5′ to the right of that of the primary rays falling on the plate. The breadth of a hump representing tertiary radiation, however, being much greater than

the breadth of a peak representing a primary line, part of the tertiary radiation shows itself beyond and to the right of the α peaks.

Table I contains the numerical data in some detail. Column 1 contains the actual readings on the spectrometer scale of the various characteristics represented on the curve. Column 2 contains the angles of incidence, θ, corresponding to the readings in Column 1. Column 3 contains the order of the spectrum and column 4, the wave-lengths, expressed in ångströms, calculated from the ordinary Bragg equation. In computing the short wave-length limits of the tertiary radiation from tungsten α and tungsten β lines, the correction for the breadth of the incident beam has been made. Column 5 contains the wave-lengths of the tungsten and molybdenum K series lines that have been measured in our laboratory and, in addition, the short wave-length limits, λ_1, of the tertiary radiation due to the tungsten α and tungsten β lines, as calculated from equation (2). The data show a *very* close agreement between observed and previously recorded wave-lengths of tungsten and molybdenum K series lines and a good agreement (less than $^1/_2\%$) between the observed and calculated values of the wave-length limits of the tertiary spectra.

In this experiment the secondary beam emergent from the molybdenum radiator at an angle of 90° from the primary beam was analyzed. At angles of 45° and 135° the short wave-length limits of the tertiary radiation humps are found experimentally to have precisely the same angular displacements from the point where the tungsten α_1 peak begins to rise as at 90°. Depending upon such factors as filtration, however, the shapes and angular positions of the maxima of the humps differ greatly. The highest points on the humps move towards larger angles (and wave-lengths) with increase in the angle between primary and secondary beams. Comparative curves will be published later.

An experiment has been performed with silver as a secondary radiator similar to that above described for molybdenum. The curve representing the data shows (*a*) peaks corresponding to the tungsten line spectrum reflected in the first, second and third orders, respectively, of the scattered radiation, (*b*) other peaks representing the fluorescent K series lines of silver, and (*c*) broad humps corresponding to the tertiary radiation produced by the photo-electrons ejected from the silver atoms by the primary rays from the tungsten target. The peaks representing the β and γ lines in the fluorescent K series of silver lie between the second and third order spectra of the tungsten lines, and those corresponding to the α_1 and α_2 fluorescent lines of silver lie between the β and γ peaks and the α_1 and α_2 peaks in the third order spectrum of tungsten. In other words, the fluorescent K series silver lines have shorter wave-lengths than those of molybdenum. This, of course, was to be expected, for silver has a higher atomic number (47) than molybdenum (42). Further, the humps corre-

sponding to the tertiary radiation from silver lie further away from the first order tungsten peaks than in the case of molybdenum. The hump representing the tertiary radiation from tungsten α lines fall in the region of the spectrum where the second order reflections of the tungsten β and tungsten γ lines lie. Its short wave-length limit lies at an angular distance of 1°29′ from the short wave-length edge of the tungsten α_1 peak. Computation from equation (2), using the wave-lengths of the tungsten α_1 line (.2086) for λ and that of the critical absorption of silver (.4850) for λ_2 gives for the wave-length shift .1574 ångström. This corresponds to an angular shift on the scale of our calcite crystal of 1°29′35″. Similarly, the short wave-length limit of the tertiary radiation due to the tungsten β line falls at an angle θ equal to 2°41′40″, whereas, theoretically, from equation (2) it should lie at 2°42′.

There is also a small prominence just to the right of the α_1 and α_2 peaks in the first order which corresponds with the tertiary radiation due to the ejection of electrons from the L levels of silver atoms by the tungsten α rays.

TABLE II

SECONDARY AND TERTIARY RADIATION FROM SILVER DUE TO PRIMARY RADIATION FROM TUNGSTEN

Wave-Lengths Are Given in Ångströms. Zero of Crystal Table Angle = 201°6′40″

CRYSTAL TABLE READING	ANGLE, θ, OF INCIDENCE	ORDER, n, OF SPECTRUM	WAVE-LENGTH $n\lambda = 2d\sin\theta$	WAVE-LENGTH KNOWN OR CALCULATED	CHARACTER OF RADIATION
199°22′30″	1°44′10″	1	.1835	.1842	Tungsten Kβ
199° 8′30″	1°58′10″	1	.2081	.2086	Tungsten Kα_1
199° 5′30″	2° 1′10″	1	.2134	.2134	Tungsten Kα_2
199° 2′0″	2° 4′30″	1	...	...	Tertiary from L Electrons
198°25′ 0″	2°41′40″	1	.296	.297	Tertiary f. Wβ
197°46′ 0″	3°20′40″	1	.365	.366	Tertiary f. Wα
197° 9′50″	3°56′50″	2	.2084	.2086	Tungsten α_1
197° 3′55″	4° 2′45″	2	.2136	.2134	Tungsten α_2
196°28′ 0″	4°38′40″	1	.4904	.491	Silver Kγ
196°22′ 0″	4°44′40″	1	.5009	.501	Silver Kβ
196° 1′30″	5° 5′10″	3	.1790	.1790	Tungsten Kγ
195°52′30″	5°14′10″	3	.1842	.1842	Tungsten Kβ
195°47′10″	5°19′30″	1	.5620	.562	Silver Kα_1
195°44′10″	5°22′30″	1	.5672	.567	Silver Kα_2
195°10′45″	5°55′55″	3	.2086	.2086	Tungsten Kα_1
195° 2′10″	6° 4′30″	3	.2136	.2134	Tungsten Kα_2

The very close agreement between the observed wave-lengths of the various characteristics of the silver curve and the wave-lengths previously observed of the tungsten and silver K series lines, together with the calculated values of the short wave-length limits of the tertiary spectra can be seen from table II. The first and second columns of this table contain the actual readings on the spectrometer scale corresponding to the various

features and the values of the glancing angle of incidence, θ, calculated from them. The third column contains the order of the spectrum and the fourth column the observed wave-lengths computed from Bragg's equation: $n\lambda = 2d \sin \theta$. The fifth and sixth columns contain, respectively, the known and recorded wave-lengths of the K series lines of tungsten and silver and the calculated wave-lengths of the short wave-length limit of the tertiary radiations, and the character of the radiation represented by each peak and hump.

The conclusions to be drawn from the experiments described in this and the two preceding notes may be summarized as follows: We have examined secondary radiators consisting of some fifteen different chemical elements whose atomic numbers range from 6 (that of carbon) to 60 (that of neodymium). The radiation from the secondary radiator produced by the primary rays consists of a continuous spectrum beginning at a short wave-length limit and extending toward the longer wave-lengths. This spectrum may be regarded as produced mainly by the primary continuous spectrum. Upon the secondary continuous spectrum are superposed peaks representing (*a*) the scattered radiation from the K series line spectra of tungsten reflected in the first, second and third order spectra. These peaks correspond to wave-lengths that are equal to those of the primary rays to within experimental error (.1%); (*b*) peaks representing the K series fluorescent line spectra of the secondary radiator, where these lie within the range of the spectrum examined; the wave-lengths corresponding to these peaks equal the wave-lengths of the primary K series lines of the respective chemical elements to within the limits of error of measurement (.1% in the case of molybdenum); (*c*) humps, the short wave-length limits of which correspond to their theoretical values, as calculated from equation (2) and which, therefore, represent tertiary radiation from electrons ejected by the primary tungsten lines. We have not observed any marked characteristics of the radiation that cannot be explained as due to the above-mentioned causes. In particular, we have not observed on any of our curves, peaks which correspond to a shift of exactly .024 ångström (an angular shift of 13 to 14′ in the position of the reflecting crystal), which should occur if the incident X-ray quanta delivered their energy and momentum to single electrons, according to Compton's theory.

[1] These PROCEEDINGS, 9, December, 1923.

FURTHER EXPERIMENTS UPON THE REFLECTION BY A CRYSTAL OF ITS CHARACTERISTIC X-RADIATION

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Communicated, December 5, 1923

During the development of a new method of analyzing crystals by X-rays we discovered that the X-rays characteristic of and excited in the chemical elements in a crystal are reflected by the crystal itself. The reflection takes place both in accordance with the Bragg law: $n\lambda = 2d \sin \theta$ and also in an anomalous fashion. The ionization spectrometer utilized in these investigations and the various steps in the method of analyzing crystals have already been described in detail.[1] From the spectra of the normally reflected X-rays characteristic of the constituent elements in the crystal we have been able to determine accurately the spatial distribution of the atoms in the crystal lattices of potassium iodide, cesium iodide, potassium tri-iodide, cesium tri-iodide and cesium dibromo-iodide, and to distinguish clearly between planes containing iodine and cesium atoms in spite of the nearly identical reflecting powers.

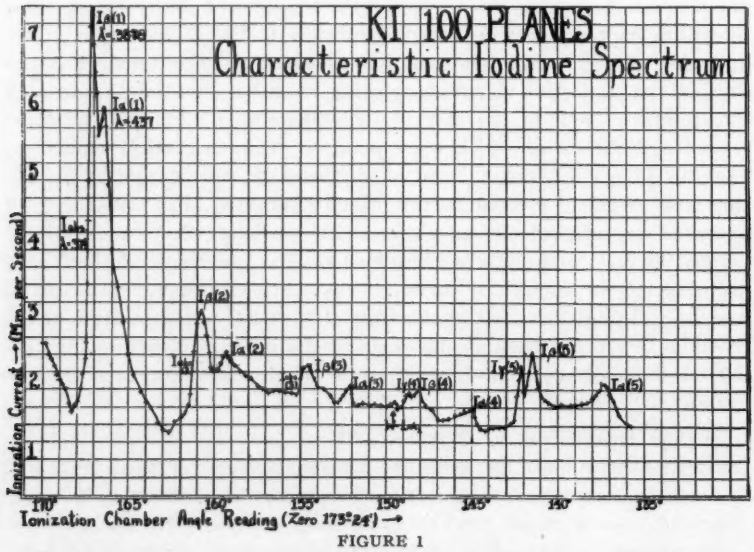

FIGURE 1

It is the purpose of the present note to present (1) a new precision measurement of the spectrum, characteristic of iodine, reflected through 5 orders

by the 100 planes of potassium iodide in accordance with the Bragg law, and (2) a further verification of the *anomalously* reflected X-rays characteristic of iodine from potassium iodide.

A crystal of pure potassium iodide, 2 mm. square and .75 mm. thick, was set up on the crystal table of the ionization spectrometer. A narrow beam of the general radiation from a tungsten target struck the crystal in such a way as to be reflected from the 100 planes into the ionization chamber. The spectrum of this reflection appears in figure 1. The mirror image of this curve has been obtained on the opposite side of the zero or direct beam zone. The angular deviations from the zero of the ionization chamber (at 173°24′) give the values of 2θ, since the abscissas refer to ionization chamber angles.

The following table presents the essential data involved in the spectrum curve in columns as follows: (1) ionization chamber angle reading; (2) the angle of incidence, θ; (3) the wave-length, λ; (4) the kind of radiation; (5) the order of the spectrum; and (6) the calculated value of θ, taking d_{100} for potassium iodide as 3.532 ångströms.

DATA RELATIVE TO THE REFLECTED IODINE RADIATION

IONIZATION CH. ANGLE	ANGLE, θ, OBSERVED	RECORDED WAVE-LENGTH	KIND OF RADIATION	ORDER OF SPECTRUM	ANGLE, θ, CALCULATED
167°20′	3° 2′	.3737	I abs.	1	3° 2′
167° 6′	3° 9′	.3878	I β	1	3° 8′50″
166°18′	3°33′	.437	I α	1	3°33′
161°15′	6° 4′1/2	.3737	I abs.	2	6° 4′30″
160°47′	6°18′1/2	.3878	I β	2	6°18′ 5″
159°12′	7° 6′	.437	I α	2	7°6′ 25″
155° 5′	9° 9′1/2	.3737	I abs.	3	9° 8′
154°27′1/2	9°28′1/4	.3878	I β	3	9°28′45″
152° 0′	10°42′	.437	I α	3	10°41′40″
149°20′	12° 2′	1.473	W $L\alpha_1$	1	12° 2′
149° 0′	12°12′	.3737	I abs.	4	12°13′
148°30′	12°27′	.3818	I γ	4	12°29′
148° 0′	12°42′	.3878	I β	4	12°40′40″
144°50′	14°17′	.437	I α	4	14°19′30″
142° 5′	15°39′1/2	.3818	I γ	5	15°40′
141°30′	15°57′	.3878	I β	5	15°55′30″
137°20′	18° 2′	.437	I α	5	18° 1′

Zero of ionization chamber = 173°24′.
Wave-lengths are expressed in ångströms.
Grating space d = 3.532 ångströms.

The experimental results presented in figure 1 and the table lead to the following interpretations: five groups of peaks appear, marked by sharp absorption drops on their short wave-length sides. The wave-length corresponding to each discontinuity has been absolutely determined by setting on each point, lowering the voltage applied to the X-ray tube in steps until the ionization current disappeared,

and calculating from the quantum equation, $Ve = h\nu$, or $\lambda = hc/Ve$. Representative critical voltages for particular points in figure 1 are 33,030 (abs. drop at 167°20′), 32,300 (peak at 142°5′), 31,800 (peak at 141°30′) and 28,250 (peak at 137°20′), respectively. These voltages correspond to the wave-lengths .3737 (I abs.), .3818 (Iγ, detected and measured for the first time), .3878 (Iβ) and .437 (Iα), all in the K series. The spectrum represents, therefore, the characteristic K series of the iodine atoms in the crystal of potassium iodide. Substituting the wave-lengths and corresponding angles in the Bragg equation, $n\lambda = 2d \sin \theta$, gives for the average value of d, 3.532 ångström units, a value identical with that previously obtained with a wave-length from the continuous spectrum. The reflection of these characteristic rays is, therefore, regular in the sense that they obey the Bragg law. On comparing columns (2) and (6) in the table it appears that any particular point will give a value of d within a small fraction of 1% of 3.532, since the agreement between θ observed and θ calculated from this value of d is very satisfactory. The slight discrepancies are not systematic indicating that the values of λ and of d are as nearly correct as may be ascertained from such experiments.

The spectrum appears only when the X-ray tube is operated above 33,000 volts; i.e., the primary beam must contain wave-lengths equal to or shorter than the critical absorption wave-length of iodine. We used 65,000 volts in obtaining the spectrum shown here. In other experiments with voltages above 69,000 volts, the tungsten line spectrum also appeared mixed with the iodine spectrum. Even at 80,000 volts, however, the latter is much more intense than the tungsten spectrum. A comparative energy study of the primary beam at 50,000 and 78,400 volts shows an enormous difference in the region of the spectrum shorter than the critical absorption wave-length of iodine, .3737 ångström—a fact which is of great importance in the production and detection of both anomalously and regularly reflected iodine rays. Curves illustrating these comparisons will be published soon.

Faint evidence of the strong $L\alpha_1$ line of tungsten at 149°20′ exists. The great intensity of the iodine spectrum and the enormous absorption of the crystal for long waves, however, practically blot out the tungsten L spectrum.

The crystal of potassium iodide may be said to be fairly opaque to rays shorter than .3737 ångström, i.e., to those reflected at angles smaller than 3°2′. In experiments with voltages above 69,000 volts the first order tungsten K-lines at about half this angle are of feeble intensity. This illustrates the possibility of serious errors in crystal structure analyses where relative intensities of spectral lines are involved, if these lines happen to have wave-lengths in the neighborhood of the critical absorptions of the elements in the crystal. Mie,[2] for example, has recently corrected such

errors by a study of these characteristic spectra, excited and reflected by crystals, that we discovered some years ago.

The characteristic iodine spectrum lines are more diffuse than the regularly reflected lines characteristic of the X-ray tube target. We have never been able, by the most careful measurements, to completely separate α, β and γ peaks in the first order. The α and β peaks are practically separated in the second order, and β and γ are separated partially in the fourth order and completely in the fifth order. The α peak is not resolved into α_1 and α_2 even in the fifth order.

The relative intensities of the various characteristic lines are quite different from those in all target and secondary fluorescent spectra. The β peak is always higher than the α, while for spectra characteristic of targets, the intensities stand in the order: $\gamma:\beta:\alpha_2:\alpha_1 = 15:35:50:100$.[3]

I—$K\gamma$ with a wave-length .3818 ångstrom is isolated and measured for the first time. It has also been identified, with the same wave-length, in the secondary fluorescent spectrum of iodine from potassium iodide due to primary radiation from a tungsten target, the secondary radiation being analyzed by a calcite crystal spectrometer. (These PROCEEDINGS, Dec. 1923.)

The relatively great intensity of the fifth order peaks is undoubtedly connected with the fact that within this angular range the anomalous reflection of iodine rays seems to coincide with the regular reflection of iodine rays from the 100 planes. A diagram of the relative positions of

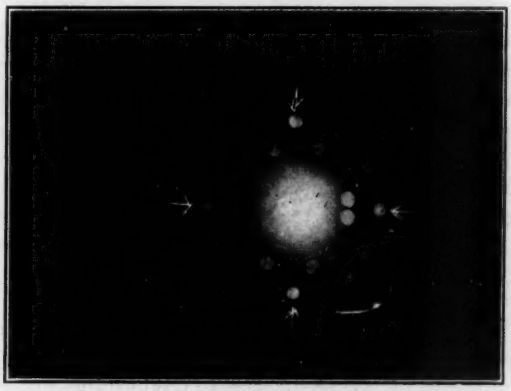

FIGURE 2

the so-called X-peaks as they depend on the angle of incidence has been published in these PROCEEDINGS, April 1923, page 133. Here the extrapolated X-peak lines intersect the line representing reflection from the

100 planes at an angle (ionization chamber) of about 32°. Table I shows that Iβ (5) appears at an equal angle, 2θ, of 31°54′.

Although one experimenter has not been able to detect this anomalously reflected characteristic radiation[4] a repetition of our experiments with the ionization spectrometer and with Laue photographs has verified completely our previous results.[5] A Laue photograph of the potassium iodide crystal described above is reproduced here (Fig. 2). The essential experimental details are as follows: one pin-hole, 1.6 mm. in diameter in a lead plate directly in front of crystal; film 3 cm. behind crystal; beam parallel to a principal axis; tungsten-target tube operated at a non-fluctuating potential difference of 78,400 volts for 8 hours; target crystal distance = 75 cm. and tube so placed that X-rays passing through pin-hole almost grazed the target's surface on leaving it. Some 60 spots appear on the photograph itself in addition to the central image of the direct beam. Most of these probably are due to regular reflection of particular wave-lengths in the general radiation by various sets of parallel planes. Four spots (indicated by arrows), which are not less intense than any others, appear above, below, right and left, on lines bisecting the photograph at right angles, each at a distance of about 1.3 cm. from the center. The angle, whose tangent is one-third of 1.3 = .4333, is about 23°30′, which agrees closely with the value of 23°36′ obtained from the diagram mentioned above. Thus, when the 100 planes are at $\theta = 0$, both the ionization chamber and the photograph register the anomalously reflected iodine rays at about 23°33′.

That the reflection is not an exact phenomenon is proven by the fact that when two pin-holes define the incident beam upon the crystal instead of one, the Laue spots of X-peaks are so greatly cut down in intensity as compared with the regular reflections that they are barely visible after 8 hours' exposure. As pointed out above, the reflection of rays characteristic of elements in a crystal by the same crystal is not exact, but somewhat diffuse.

There are some features of the Laue photographs which are difficult to explain. Without definite experimental knowledge of the wave-lengths producing given spots, such as is possible in our ionization spectrometer method, caution must be exercised in interpreting Laue photographs as a method of studying crystal structures. Wave-lengths have sometimes been assigned to spots which are actually not present in the primary beam because of an operating voltage too low to produce those wave-lengths at all. Furthermore, it is often necessary to use a voltage considerably above a critical value in order to give sufficient intensity for photographic analysis. Thus, though the critical voltage for the X-spots is 33,000, or that required to produce the critical absorption wave-length of iodine (.3737 ångström), 75,000 volts produces a beam very much richer in rays with

this and shorter wave-lengths and, consequently, the effect is much greater. Since our discovery of the anomalously reflected iodine rays from potassium iodide, several cases of anomalous reflection have been reported and explanations suggested by McKeehan[6] on several metals, Dickinson[7] on potassium iodide and tin tetra-iodide, and Mie[2] on bismuth, etc. Apparently, the explanations are not adequate, however, for the particular cases discussed here.

The excitation of X-rays characteristic of constituent elements and their regular reflection by the same crystal seems to be reasonably well explained on the basis of the principle of the transfer of radiation momenta in quanta.[8] This analysis alone accounts for the facts, including the peculiar relative intensities, the diffuse nature of the lines, etc. Mie[2] considers the phenomenon as true optical resonance. If this is true, it should be possible, by using a voltage between 28,250 and 31,800, to produce an α peak without a β or γ peak. There is every reason to believe from our previous researches that the α line cannot be produced without the β and γ, just as it cannot be produced alone by the target. Furthermore, if spherical characteristic wave-trains spreading out from atoms in the crystal and reflected from other atoms produced these effects, then they ought not to depend much upon the angle of incidence of the primary X-ray, which they do, according to our experiments.

[1] Clark and Duane, These PROCEEDINGS, **8**, 90 (1922); **9**, 117, 126, 131 (1923); *J. Optical Soc. Amer.*, **7**, 455 (1923); *Science*, **58**, 398 (Nov. 13, 1923).

[2] *Zeit. Physik*, **15**, 56; **18**, 105 (1923).

[3] Duane and Stenström, these PROCEEDINGS, **6**, 477 (1920).

[4] Wyckoff, *Science*, July 20, 1923.

[5] Clark and Duane, *Science*, Nov. 13, 1923.

[6] McKeehan, *J. Optical Soc. Amer.*, **6**, 989 (1922).

[7] Dickinson, *Phys. Rev.*, Aug. 1923, p. 199.

[8] Duane, these PROCEEDINGS, **9**, 158 (1923).

THE LAW OF SUN-SPOT POLARITY

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In a paper on "The Direction of Rotation of Sun-spot Vortices," published in the *Proceedings of the National Academy of Sciences* in June, 1915, I described the general reversal of the magnetic polarity of sun-spots observed at the minimum of solar activity in 1912. After the usual interval of about eleven years another minimum has occurred, and the incoming

spots of the new cycle following it show another reversal of polarity. It therefore becomes possible to formulate a polarity law based on observations by Hale, Ellerman, Nicholson, Joy, Pettit and others of the Zeeman effect in 2136 sun-spot groups during the period 1908–1923.*

About 60 per cent of all sun-spots are bipolar, consisting of two spots or groups of spots of opposite magnetic polarity. In classifying the observations we give chief weight to these, and treat single spots followed by flocculi as the preceding members of incomplete bipolar groups, and single spots preceded by flocculi as the following members of such groups. Before the minimum of 1912 the polarity of preceding spots in the northern hemisphere was S (south seeking pole) or negative, and that of following spots N or positive. The spots of the southern hemisphere gave opposite polarities, i.e., N for the preceding and S for the following members.

As is well known, the last spots of a cycle occur in low latitudes, ranging from 0° to about 18°. The first spots of a new cycle appear in high lati-

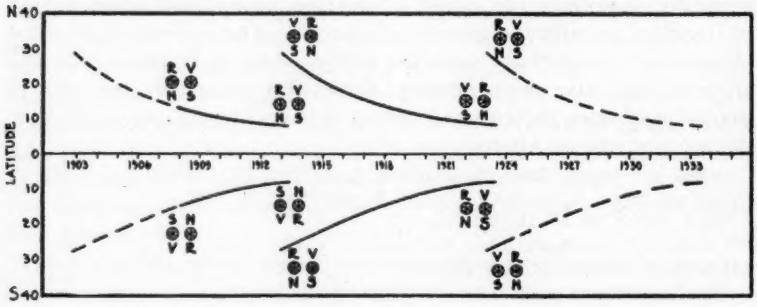

FIGURE 1

tudes, sometimes exceeding 40°. As the cycle progresses, the average latitude of all spots decreases steadily, as shown by the curves in Fig. 1.

To our surprise, the high latitude spots of the new cycle, which began in 1912 when spots were very few, were opposite in polarity to the low latitude spots of the previous cycle. As this cycle advanced and the spots became more and more numerous, the new polarities were found to characterize all spots observed, with only 4 per cent of exceptions. The average latitude of the spots gradually decreased, in the customary way, and the recent spots marking the end of the cycle have been near the equator, though several of them observed in 1923 reached latitudes as high as 15°.

The first spot of the next (third) cycle appeared on June 24, 1922, at latitude 31° N. After a long interval several other spots of this cycle have been observed, including a number of bipolar groups. They again show a reversal of polarity, back to the conditions existing in the first cycle before the minimum of 1913. The results are shown graphically in Figs. 1 and 2.

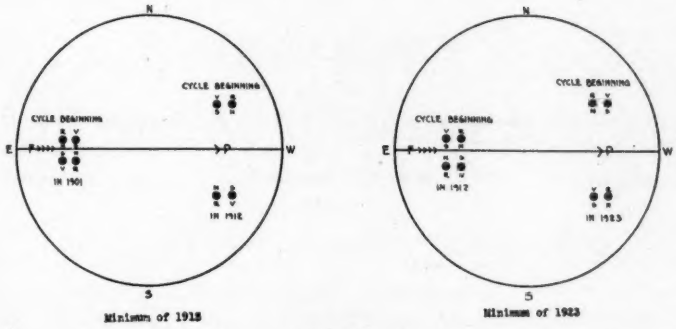

FIGURE 2

It thus appears that near the time of minimum solar activity four spot zones, characterized by distinct magnetic polarities, may co-exist on the sun. This condition lasts only two or three years, after which the last low-latitude spots of the old cycle disappear.

We cannot yet determine with certainty the sign of the dominant electric charge in the spot vortex. If it is always the same, the vortices of the preceding and following spots of bipolar groups must whirl in opposite directions. Moreover, the reversed polarities of corresponding spots (preceding or following) in the same hemisphere of successive cycles must also mean opposite directions of whirl. A series of observations of the Evershed effect at low levels in bipolar spots is needed to settle this question and thus to determine the sign of the dominant charge.

The sun-spot period, if defined in the usual way as representing the variation in the number or total area of all spots on the sun, is about 11.1 years. But if we regard the period as the interval between successive appearances of spots of the same magnetic polarity, the present results indicate that it is twice as long.

The details of this investigation will be published in the *Astrophysical Journal*.

* For the methods employed, see "The Magnetic Polarity of Sun-spots," Contributions from the Mount Wilson Observatory No. 165. *Astrophys. J., Chicago*, **49**, 1919 (153–178).